

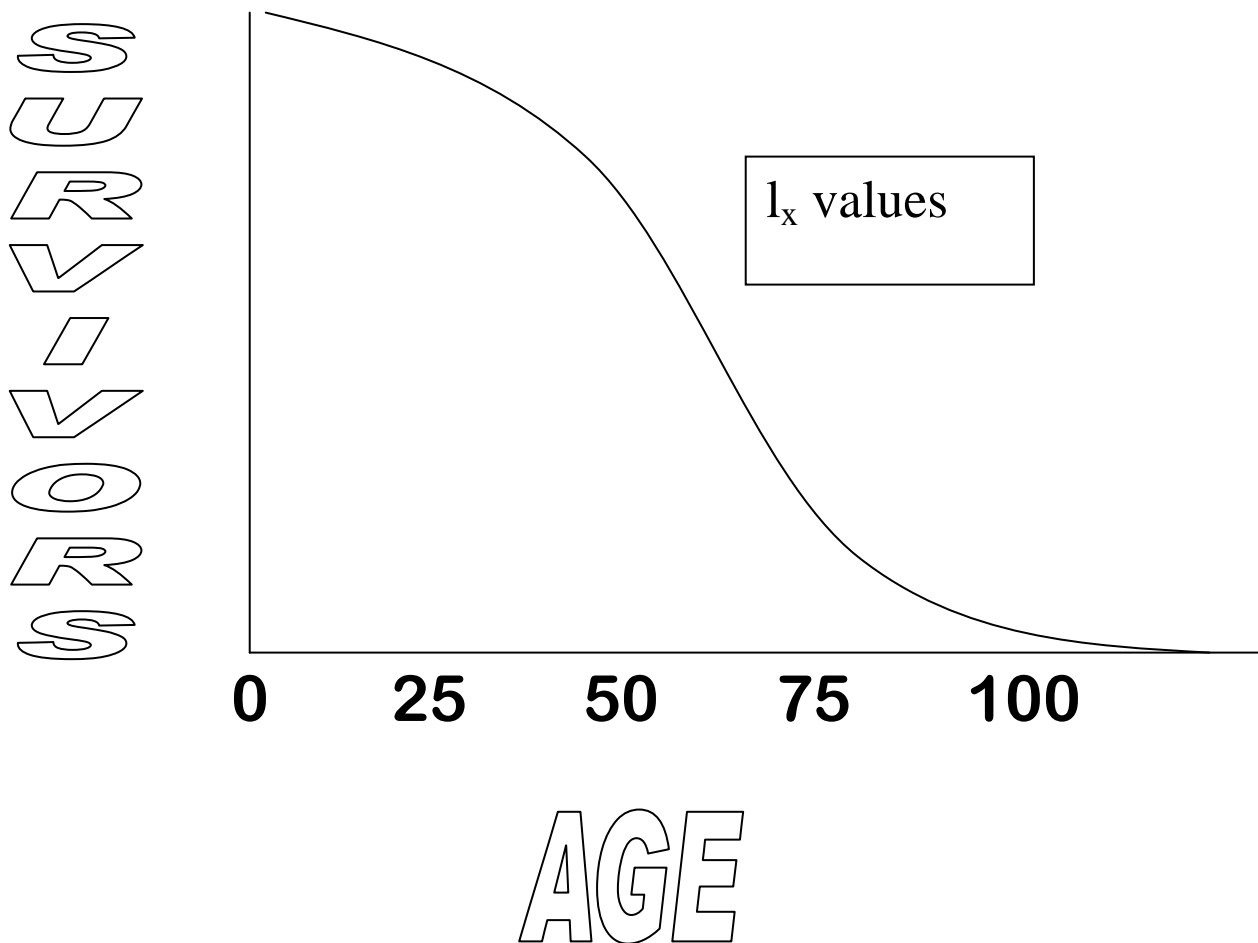
EA-1 SEMINAR

SECTION 13

POPULATION THEORY

LIFE CONTINGENCIES

Graph of l_x curve



STATIONARY POPULATION THEORY

BASIC DEFINITIONS

Stationary population remains constant over time

New entrants replace all exits, every year

STATIONARY POPULATION THEORY

BASIC DEFINITIONS

$$l_x = \int l_{x+t} \mu_{x+t} dt$$

= # people attaining age x in a year

= # people dying at age x and over
in a year

$$l_x = d_x + d_{x+1} + d_{x+2} + \dots$$

STATIONARY POPULATION THEORY

BASIC DEFINITIONS

$$T_x = \int l_{x+t} dt$$

= # people now age x and over

$$T_x = \int t l_{x+t} \mu_{x+t} dt$$

= # years lived from age x to death by
the l_x people attaining age x in a year

STATIONARY POPULATION THEORY

KEY QUANTITIES

$$l_x - l_{x+n} = d_x + d_{x+1} + d_{x+1} + \dots + d_{x+n-1}$$

= Number of people who die between age x and age x+n in a year

$$T_x - T_{x+n} - n l_{x+n}$$

= Number of people now between age x and age x+n who will die before age x+n

= Total lifetime, from age x until death, of those who die between age x and age x+n in a year

STATIONARY POPULATION THEORY

AVERAGE AGE AT DEATH

$$\frac{T_x}{l_x} = {}^{\circ}e_x$$

$$\frac{T_0}{l_0} = {}^{\circ}e_0$$

= Average age at death for original group
of l_0 lives

STATIONARY POPULATION THEORY

AVERAGE AGE AT DEATH

$$\frac{x l_x + T_x}{l_x} = x + \frac{T_x}{l_x}$$

= total past lifetime + total future lifetime
divided by number of lives at age x

= Average age at death of those already
age x

STATIONARY POPULATION THEORY

AVERAGE AGE AT DEATH

$$x + \frac{T_x - T_{x+n} - n l_{x+n}}{l_x - l_{x+n}}$$

= Average age at death of those who die
between age x and age x+n

STATIONARY POPULATION THEORY

AVERAGE AGE

$$Y_x = \int \int t l_{y+t} \mu_{y+t} dt dy$$

= total future lifetime of people now age x and over

$$Y_x = \int \int (y-x) l_{y+t} \mu_{y+t} dt dy$$

= total past lifetime, since age x, of people now age x and over

STATIONARY POPULATION THEORY

AVERAGE AGE

$$Y_x - Y_{x+n} - n T_{x+n}$$

= Total past lifetime, since age x, of
people now between age x and age x+n

= Total future lifetime, before age x+n, of
those now between age x and age x+n

$$x + \frac{Y_x - Y_{x+n} - n T_{x+n}}{T_x - T_{x+n}}$$

= Average age for those members
between age x and age x+n

STATIONARY POPULATION THEORY

AVERAGE AGE AT DEATH

$$x T_x + Y_x + Y_x$$

For the T_x people at age x and over, their past lifetime before age x is $x T_x$, their total past lifetime after age x is Y_x , and their total future lifetime is Y_x .

$$\frac{x T_x + 2Y_x}{T_x}$$

= Average age at death of those who die at age x and over

STATIONARY POPULATION THEORY

AVERAGE AGE AT DEATH

$$xT_x + 2Y_x$$

Total lifetime for the T_x people at age x and over

$$(x+n)T_{x+n} + 2Y_{x+n}$$

Total lifetime for the T_{x+n} people at age $x+n$ and over

STATIONARY POPULATION THEORY

AVERAGE AGE AT DEATH

$$xT_x + 2Y_x - \{(x+n)T_{x+n} - 2Y_{x+n}\}$$

Total lifetime for the $T_x - T_{x+n}$ people
between ages x and $x+n$

$$\frac{xT_x + 2Y_x - \{(x+n)T_{x+n} - 2Y_{x+n}\}}{T_x - T_{x+n}}$$

= Average age at death for those
members between age x and age $x+n$

SECTION XIII - POPULATION THEORY

1. Number dying in year between ages x and $x+n$:

$$l_x - l_{x+n}$$

2. Total Population:

$$T_x - T_{x+n}$$

3. Average age at death for the deaths between ages x and $x+n$:

$$x + \frac{T_x - T_{x+n} - n l_{x+n}}{l_x - l_{x+n}}$$

4. Expectation of life:

$$e_x^{\circ} = \frac{T_x}{l_x}$$

5. Change in population resulting from change in birth rate

OPENING / CLOSING COMMENTS

READ	Overheads - key background for each exam topic area
WORK	Prior exam problems
EXPECT	Similar problems as last 2 years
STUDY	"New stuff" in last 2 years' exams
REVIEW	Lengthy exam solutions give background of WHY - not just HOW to solve problem
NEXT STEPS	PRACTICE, PRACTICE, ETC. Be speedy in working problems
EMAIL	Follow-up questions, clarifications after the seminar