SECTION 12

MULTIPLE DECREMENTS

MULTIPLE DECREMENT TABLES

Active service table allows for multiple decrement probabilities:

$$p_x^{(T)} = 1 - q_x^{(1)} - q_x^{(2)}$$

Pension valuations typically have four decrements:

Termination
Retirement
Mortality
Disability

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MULTIPLE DECREMENT TABLES

Active service table allows for multiple decrement probabilities.

These probabilities are derived based on rates of decrement from associated single decrement tables:

$$p'^{(1)}_x = 1 - q'^{(1)}_x$$

$$p'^{(2)}_x = 1 - q'^{(2)}_x$$

$$p^{(T)}_x = p'^{(1)}_x \, p'^{(2)}_x$$

$$= [1 - q'^{(1)}_x][1 - q'^{(2)}_x]$$

SINGLE DECREMENT TABLES

Do they really exist?

Except for mortality, single decrement tables are just a theoretical construct.

MULTIPLE DECREMENT TABLES

Approximate relationships between multiple decrement probabilities and rates of decrement:

$$q'^{(1)}_x \approx q^{(1)}_x / [1 - \tfrac{1}{2}q^{(2)}_x]$$

$$q^{(1)}_x \approx q'^{(1)}_x (1 - \tfrac{1}{2}q'^{(2)}_x) / (1 - \tfrac{1}{4}\, q'^{(1)}_x q'^{(2)}_x)$$

Assumes two sources of decrement, and uniform distribution of decrements in the multiple decrement table.

Another common approximation:

$$q'^{(1)}_x \approx q^{(1)}_x * [1 + \tfrac{1}{2}q^{(2)}_x]$$

MULTIPLE DECREMENT TABLES

Bowers et al shows exact formulas, for <u>any</u> number of decrements:

$$ {}_t p'^{(1)}_x = \left[{}_t p^{(T)}_x \right]^{\frac{q^{(1)}_x}{q^{(T)}_x}} $$

- Assumes constant force for multiple decrements (10.5.10) or uniform distribution of multiple decrements (10.5.12) for a single year of age.
- These may be more convenient to use than prior formulas.
- Small difference in results using the different formulas.

MULTIPLE DECREMENT TABLES

Bowers et al shows exact formulas, for <u>any</u> number of decrements:

Force of decrement:

$$\mu_{x+t}^{(\tau)} = \sum_{j=1}^{m} \mu_{x+t}^{(j)} \qquad \textbf{10.2.14}$$

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MULTIPLE DECREMENT TABLES

If uniform distribution of decrements in <u>single</u> decrement tables, and two decrements, then

$$q'^{(1)}_x \approx q^{(1)}_x / [1 - ½\, q'^{(2)}_x] \qquad \textbf{based on 10.6.3}$$

PROOF OF BOWERS 10.6.3

$$q_x^{(1)} = \int_0^1 {}_tp_x^{(T)} \mu_{x+t}^{(1)} dt$$

$$= \int_0^1 {}_tp_x'^{(2)} \, {}_tp_x'^{(1)} \mu_{x+t}^{(1)} dt$$

Based on page 91, under U.D.D.

$$q_x'^{(1)} = {}_tp_x'^{(1)} \mu_{x+t}^{(1)}$$

$$q_x^{(1)} = q_x'^{(1)} \int_0^1 {}_tp_x'^{(2)} dt$$

$$= q_x'^{(1)} \int_0^1 1 - tq_x'^{(2)} dt$$

$$= q_x'^{(1)} \left[t - \frac{t^2}{2} q_x'^{(2)} \right]_0^1$$

$$= q_x'^{(1)} \left[1 - \tfrac{1}{2} q_x'^{(2)}\right]$$

CENTRAL RATE OF DECREMENT

Bowers et al shows various formulas:

Central rate from all causes

$$m_x^{(\tau)} = \frac{\int_0^1 {}_tp_x^{(\tau)}\,\mu_{x+t}^{(\tau)}\,dt}{\int_0^1 {}_tp_x^{(\tau)}\,dt} \qquad 10.5.5$$

Central rate from cause j

$$m_x^{(j)} = \frac{\int_0^1 {}_tp_x^{(\tau)}\,\mu_{x+t}^{(j)}\,dt}{\int_0^1 {}_tp_x^{(\tau)}\,dt} \qquad 10.5.6$$

Logically, you have $m_x^{(\tau)} = \sum_{j=1}^{m} m_x^{(j)}$

CENTRAL RATE OF DECREMENT

Bowers et al shows various formulas:

Central rate for single decrement table

$$m'^{(j)}_x = \frac{\int_0^1 {}_tp'^{(j)}_x \, \mu^{(j)}_{x+t} \, dt}{\int_0^1 {}_tp'^{(j)}_x \, dt} \qquad 10.5.7$$

CENTRAL RATE OF DECREMENT

From Exercise 10.18, two other formulas:

If uniform distribution of decrements in single decrement tables, then

$m'^{(j)}_x \approx q'^{(j)}_x \;/\; [1 - ½\, q'^{(j)}_x]$

If uniform distribution of decrements in multiple decrement tables, then

$q^{(j)}_x \approx m^{(j)}_x \;/\; [1 + ½\, m^{(T)}_x]$

FORCE OF MORTALITY

Force of mortality: measures mortality at exact age x, expressed as annual rate

Single decrement table definition:

$$\mu_x = \frac{-1}{l_x}\left[\frac{d}{dx} l_x\right]$$

FORCE OF DECREMENT

Multiple decrement table definition:

$$\mu_x^{(k)} = \frac{-1}{l_x^{(T)}}\left[\frac{d}{dx} l_x^{(k)}\right]$$

From page 191:

$$\mu_{x+t}^{(T)} = \sum_{j=1}^{m} \mu_{x+t}^{(j)} \qquad 10.2.14$$

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KEY CONCEPTS

SECTION XI – MULTIPLE DECREMENTS

1. Rates versus Probabilities
   a. Probability: $p_x^{(1)}$, $q_x^{(1)}$
   b. Rate: $p_x'^{(1)}$, $q_x'^{(1)}$
2. Approximations - UDD in multiple decrement table

   $q_x'^{(1)} \approx q_x^{(1)} / [1 - \frac{1}{2} q_x^{(2)}]$

   $q_x^{(1)} \approx q_x'^{(1)} (1 - \frac{1}{2} q_x'^{(2)}) / (1 - \frac{1}{4} q_x'^{(1)} q_x'^{(2)})$
3. Exact formula (UDD / constant force)

   $${}_t p_x'^{(1)} = \left[{}_t p_x^{(T)} \right]^{\frac{q_x^{(1)}}{q_x^{(T)}}}$$
4. Approximation - UDD in single decrement table

   $q_x'^{(1)} \approx q_x^{(1)} / [1 - \frac{1}{2} q_x'^{(2)}]$
5. Central rate of decrement

   $m_x'^{(j)} \approx q_x'^{(j)} / [1 - \frac{1}{2} q_x'^{(j)}]$
6. Force of decrement

   $$\mu_x^{(k)} = \frac{-1}{l_x^{(T)}} \left[\frac{d}{dx} l_x^{(k)} \right]$$

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