SECTION 9

INSURANCE VALUES

COMMUTATION FUNCTIONS

In real world, actuaries use commutation functions to simplify calculations. Computer calculations are faster with commutation functions.

$C_x = v^{x+1}(d_x)$

$M_x = \sum_{t=0}^{\omega - x} C_{x+t}$

$R_x = \sum_{t=0}^{\omega - x} M_{x+t}$

NOTE

Since 2000, the EA exams rarely tested anything other than Nx and Dx.

COMMUTATION FUNCTIONS

Easy to value payments upon death:

$C_x = v^{x+1}(d_x)$
Interpret C_x as single payment, payable at end of year of death, upon death "at age x"

$M_x = \sum_{t=0}^{\omega-x} C_{x+t}$
Interpret M_x as payment, payable at end of year of death, upon death after attaining age x

$R_x = \sum_{t=0}^{\omega-x} M_{x+t}$
Interpret R_x as increasing payment, payable at end of year of death, upon death after attaining age x.

Payment is 1 for death "at age x", 2 for death "at age x+1", and 1+t for death "at age x+t"

INSURANCE DEFINITIONS

A_x

Whole Life Insurance pays 1 at the end of the year of death.

$A^{1}_{x:\overline{n|}}$

Term Insurance pays 1 at the end of the year of death if you die before age x+n

$A_{x:\overline{n|}}$

Endowment Insurance pays 1 at the end of the year of death if you die before age x+n. It also pays 1 at the end of n years if you survive to age x+n.

LIFE INSURANCE

$$A_x = v^1 \, {}_{0|}q_x + v^2 \, {}_{1|}q_x + \ldots + v^n \, {}_{n-1|}q_x + \ldots$$

$$= \sum_{t=0}^{\omega - x} v^{t+1} \, {}_{t|}q_x$$

$$= \frac{1}{l_x} \sum_{t=0}^{\omega - x} v^{t+1} d_{x+t}$$

$A_x = 1 - d\ddot{a}_x$

$A_x = v\ddot{a}_x - a_x$

Verbal interpretation of identities

DeMoivre's law

$l_x = \omega - x$

$d_x = 1$ at every age

Let $n = \omega - x$

$$a_x = \frac{n - \ddot{a}_{\overline{n}|i}}{ni}$$

$$\begin{aligned} A_x &= 1 - d\ddot{a}_x \\ &= 1 - iv(1 + a_x) \\ &= 1 - iv\left(1 + \frac{n - \ddot{a}_{\overline{n}|i}}{ni}\right) \\ &= (1/n)\ddot{a}_{\overline{n}|i} \end{aligned}$$

Derivation - exercise for the student!

TERM / DEFERRED INSURANCE

${}_{n|}A_x$ = Deferred Life Insurance

$= v^n({}_np_x)\, A_{x+n}$

$A^1_{x:\overline{n}|}$ = Term Insurance

$= v^1\, {}_{0|}q_x + v^2\, {}_{1|}q_x + \ldots + v^n\, {}_{n-1|}q_x$

$= A_x - {}_{n|}A_x$

ENDOWMENT INSURANCE

$A_{x:\overset{1}{\overline{n|}}}$ = Pure Endowment Insurance

$= v^n({}_np_x)$

$A_{x:\overline{n|}}$ = Term + Pure Endowment

$= A_x - {}_{n|}A_x + v^n({}_np_x)$

CONTINUOUS INSURANCE

Insurance is payable at the moment of death, instead of at the end of the year of death

$$\bar{A}_x = \int_0^\infty v^t \, {}_t p_x \, \mu_{x+t} \, dt$$
$$= 1 - \delta \bar{a}_x$$

INSURANCE PREMIUMS

General rule:
PV of premiums = PV of insurance

Whole life insurance policy
Premiums paid for life:

$A_x = P_x * \ddot{a}_x$
$P_x = A_x / \ddot{a}_x$

INSURANCE PREMIUMS

$$P_x = A_x / \ddot{a}_x$$
$$= (1 - d\ddot{a}_x) / \ddot{a}_x$$
$$= 1/\ddot{a}_x - d$$

$$P_x + d = 1/\ddot{a}_x$$

INSURANCE PREMIUMS

Term life insurance policy
Premiums paid for term of policy:

$$A^{1}_{x:\overline{n}|} = P^{1}_{x:\overline{n}|} * \ddot{a}_{x:\overline{n}|}$$

$$P^{1}_{x:\overline{n}|} = A^{1}_{x:\overline{n}|} / \ddot{a}_{x:\overline{n}|}$$

INSURANCE PREMIUMS

Endowment insurance policy

$$A_{x:\overline{n}|} = P_{x:\overline{n}|} * \ddot{a}_{x:\overline{n}|}$$

$$P_{x:\overline{n}|} = A_{x:\overline{n}|} / \ddot{a}_{x:\overline{n}|}$$

$$A_{x:\overline{n}|} = 1 - d * \ddot{a}_{x:\overline{n}|}$$

$$P_{x:\overline{n}|} = 1/\ddot{a}_{x:\overline{n}|} - d$$

GROSS PREMIUMS

Prior discussion of net premiums ignored expenses:
PV of premiums = PV of insurance

Gross premium takes into account expenses over life of policy:
PV of gross premiums = PV of insurance plus PV of all expenses

GROSS PREMIUM EXAMPLE

Whole life insurance policy
Premiums payable for n years

Policy expenses:
60% of first year premium
10% of renewal premiums
$10 per year

GROSS PREMIUM EXAMPLE

PV of gross premiums:

$G^*\ddot{a}_{x:\overline{n}|}$

PV of benefits plus expenses:

$A_x + 60\%(G) + 10\%(G)a_{x:\overline{n-1}|} + 10\ddot{a}_x$

$$G^*\ddot{a}_{x:\overline{n}|} = A_x + .5G + .1G\ \ddot{a}_{x:\overline{n}|} + 10\ddot{a}_x$$

$$.9(G)\ddot{a}_{x:\overline{n}|} - .5G = A_x + 10\ddot{a}_x$$

$$G = \frac{A_x + 10\ddot{a}_x}{.9\ddot{a}_{x:\overline{n}|} - .5}$$

INSURANCE RESERVES

Analogous to O/S balance of loan. Reserve gives net liability for an insurance policy.

At any point in time, reserve can be calculated two ways

Prospective:
Reserve = PV future less PV future
Insurance premiums
benefits

Retrospective:
Reserve = Accumulated value of past premiums minus accumulated value of past insurance benefits

INSURANCE RESERVES PROSPECTIVE FORMULA

$$ {}_{t}V_{x} = A_{x+t} - P_{x}(\ddot{a}_{x+t}) $$

$$ A_{x+t} = 1 - d\,\ddot{a}_{x+t} $$

INSURANCE RESERVES
RETROSPECTIVE FORMULA

Retrospective:

Reserve = Accumulated value of past premiums minus accumulated value of past insurance benefits

$${}_tV_x = P_x(\ddot{s}_{x:\overline{t}|}) - {}_tk_x$$

INSURANCE RESERVES

$${}_tV_x = (1 - d\,\ddot{a}_{x+t}) - P_x(\ddot{a}_{x+t})$$
$$= 1 - (P_x + d)\ddot{a}_{x+t}$$

$$P_x + d = (1 / \ddot{a}_x)$$

$${}_tV_x = 1 - (1 / \ddot{a}_x)\ddot{a}_{x+t}$$
$$= 1 - (\ddot{a}_{x+t} / \ddot{a}_x)$$

ONE YEAR TERM COST PENSION PLAN FUNDING

- Used in lieu of level normal cost
- Simple approximation
- One year at a time
- PV of liability for one year's expected exits

Example calculation:
Death benefit to survivor is monthly annuity

$v^1 \, q_x^{(d)}$ (Annual benefit) $\ddot{a}_{y+1}^{(12)}$

KEY CONCEPTS

SECTION IX - INSURANCE

1. Definitions
   a. PV premiums = PV Bens
   b. Endowment, term,
      whole life insurance

2. Insurance identities

3. Interpretation of complex word problems

4. PV of Gross premiums
   = PV Expenses + PV Bens

5. Insurance Reserves
   a. Prospective
   b. Retrospective

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