SECTION 11

ACTUARIAL EQUIVALENCE

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CERTAIN AND LIFE ANNUITY

Death benefit in first n years is continuation of benefits to end of n years.

Assume reduced benefit B is actuarially equivalent to life annuity of $1 per year:

PV of Normal Form	**=**	**PV of Optional form**

$$1^*\ddot{a}_x = B^*(\ddot{a}_{\overline{n|}} + {}_{n|}\ddot{a}_x)$$

$$= B^*(\ddot{a}_{\overline{n|}} + v^n {}_n p_x \ddot{a}_{x+n})$$

Reduction factor:

$$B = \ddot{a}_x / (\ddot{a}_{\overline{n|}} + {}_{n|}\ddot{a}_x)$$

ACTUARIALLY REDUCED EARLY RETIREMENT BENEFITS

Assume reduced benefit B payable at age X is actuarially equivalent to annual life annuity of $1 commencing at RA:

PV of Normal Form	**=**	**PV of Optional form**

$$\frac{D_{RA}}{D_X}\,\ddot{a}_{RA} = B\,(\ddot{a}_X)$$

$$B = \frac{D_{RA}\,\ddot{a}_{RA}}{D_X\,\ddot{a}_X}$$

$$= N_{RA} / N_X$$

$$= v^{RA-X}\,{}_{RA-X}p_X\,(\ddot{a}_{RA} / \ddot{a}_X)$$

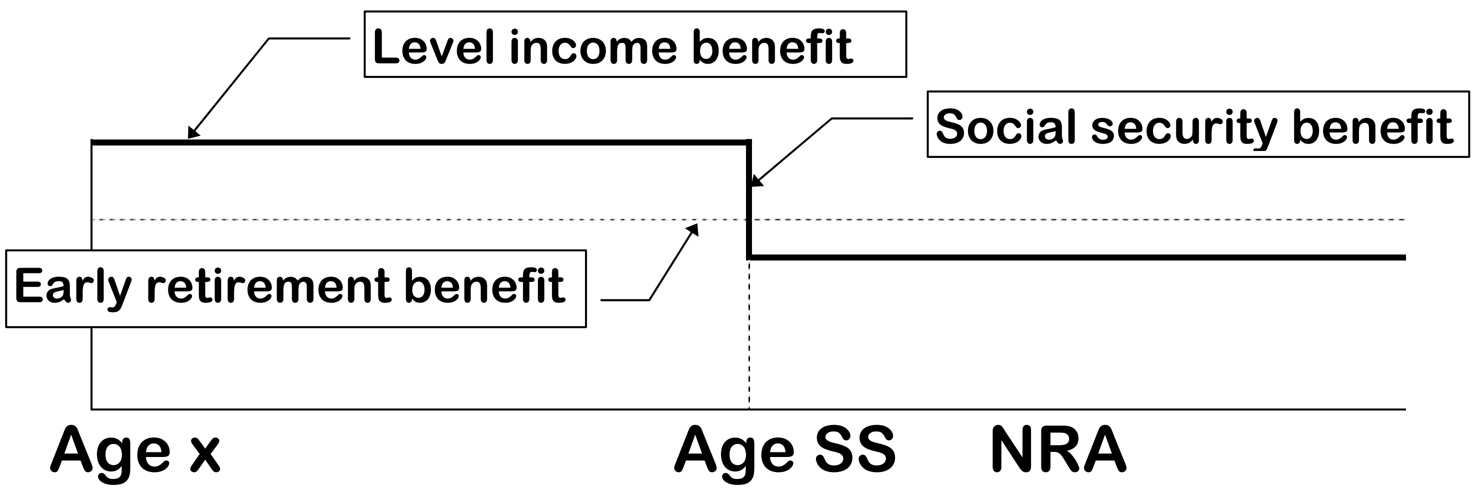
ACTUARIALLY REDUCED EARLY RETIREMENT BENEFITS

Actuarial reduction consists of three components that reflect effect of early commencement of benefits:

1. Interest lost on pension fund assets
2. Lost mortality gains
3. More payments to annuitant

LEVEL INCOME BENEFITS

Provide benefit from pension plan that is
1. Non-level
2. Actuarially equivalent to normal form
3. Produces level benefit when added to expected Social Security benefit



LEVEL INCOME BENEFITS

$$ERB\,\ddot{a}_x = LIB^*\ddot{a}_{x:\overline{ss-x}|} + (LIB - PIA)^*{}_{ss-x|}\ddot{a}_x$$

$$= LIB(\ddot{a}_x) - PIA\,(v^{ss-x}{}_{ss-x}p_x\,\ddot{a}_{ss})$$

$$LIB = ERB + PIA\,(v^{ss-x}{}_{ss-x}p_x\,\frac{\ddot{a}_{ss}}{\ddot{a}_x})$$

<u>EXTRA CREDIT</u>

Some pension plans require that
LIB - PIA > 0, otherwise can't elect LIB

EFFECT OF ASSUMPTION CHANGES

EXTRA CREDIT

	Increase i	Decrease i
INTEREST RATE	$\ddot{a}_x \downarrow$	$\ddot{a}_x \uparrow$
"J&S" factor	$\uparrow$	$\downarrow$
Early ret factor	$\downarrow$	$\uparrow$
C&L factor	?	?

- General rule: factors change in same direction as effect on $\ddot{a}_x$
- Factors as defined previously
- "Hard analysis" versus "Warm fuzzies"

EFFECT OF ASSUMPTION CHANGES

EXTRA CREDIT

	Increase q_x	Decrease q_x
MORTALITY TABLE	$\ddot{a}_x \downarrow$	$\ddot{a}_x \uparrow$
"J&S" factor	$\downarrow$	$\uparrow$
Early ret factor	$\downarrow$	$\uparrow$
C&L factor	$\downarrow$	$\uparrow$

- General rule: factors change in same direction as effect on $\ddot{a}_x$
- Factors as defined previously
- "Hard analysis" versus "Warm fuzzies"

ACTUARIALLY REDUCED EARLY RETIREMENT FACTOR

EXTRA CREDIT

$\frac{N_{RA}}{N_X}$ factor is same for early or late retirement, assume X < RA for now

$$\frac{N_{RA}}{N_X} = \left[\frac{N_{RA}}{N_{RA-1}} \cdot \frac{N_{RA-1}}{N_{RA-2}} \cdots \frac{N_{X+2}}{N_{X+1}} \cdot \frac{N_{X+1}}{N_X}\right]$$

$$\begin{aligned}\frac{N_{X+1}}{N_X} &= \frac{N_X - D_X}{N_X} \\ &= 1 - \frac{D_X}{N_X} \\ &= 1 - \frac{1}{\ddot{a}_X}\end{aligned}$$

ACTUARIALLY REDUCED EARLY RETIREMENT FACTOR

EXTRA CREDIT

$\frac{N_{RA}}{N_x}$ factor for early retirement

$$\frac{N_{x+1}}{N_x} = 1 - \frac{1}{\ddot{a}_x}$$

Effect on:	$\ddot{a}_x$,	$\frac{1}{\ddot{a}_x}$,	$1 - \frac{1}{\ddot{a}_x}$
Interest $\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$

Change in early retirement reduction factor follows effect on annuity

For late retirement increase factor, change is opposite of effect on annuity

$\frac{N_x}{N_{x+1}}$

ACTUARIALLY REDUCED CERTAIN AND LIFE FACTOR

EXTRA CREDIT

$$\ddot{a}_x \,/\, (\ddot{a}_{\overline{n}|} + {}_{n|}\ddot{a}_x)$$

$$= \ddot{a}_x \,/\, (\ddot{a}_{\overline{n}|} + \ddot{a}_x - \ddot{a}_{x:\overline{n}|})$$

$$= \ddot{a}_x \,/\, (\ddot{a}_x + \ddot{a}_{\overline{n}|} - \ddot{a}_{x:\overline{n}|})$$

$$= 1 \,/\, [1 + (\ddot{a}_{\overline{n}|} - \ddot{a}_{x:\overline{n}|}) \,/\, \ddot{a}_x]$$

$$= \left[1 + \frac{(\ddot{a}_{\overline{n}|} - \ddot{a}_{x:\overline{n}|})}{({}_{n|}\ddot{a}_x + \ddot{a}_{x:\overline{n}|})}\right]^{-1}$$

Effect on:	$\ddot{a}_{\overline{n}|}$	$\ddot{a}_{x:\overline{n}|}$	${}_{n|}\ddot{a}_x$
as $q_x \uparrow$	N/A	$\downarrow$	$\downarrow$

Final effect still unclear – next page shows proof that effect on reduction factor follows change in q_x

EXTRA CREDIT

Rewrite ratio as

$$\left(1 + \frac{(a - b)}{(c + b)} \right)^{-1} = (1 + R)^{-1}$$

After change in q_x, we have

$a' = a, \quad b' = b - \Delta b, \quad c' = c - \Delta c$

$$\left(1 + \frac{(a' - b')}{(c' + b')} \right)^{-1} = (1 + R')^{-1}$$

$$= \left(1 + \frac{(a' - b + \Delta b)}{(c - \Delta c + b - \Delta b)} \right)^{-1}$$

$$= \left(1 + \frac{(a' - b + z1)}{(c + b - z2)} \right)^{-1}$$

$(1 + R') > (1 + R)$

$(1 + R')^{-1} < (1 + R)^{-1}$

After change in q_x, we have lower factor

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