BONDS – YIELD RATES

$$P = Fr(a_{\overline{n}|i}) + K$$
$$= C + (Fr - Ci)\,a_{\overline{n}|i} \qquad g = \frac{Fr}{C}$$
$$= C + C(g - i)\,a_{\overline{n}|i}$$
$$\frac{P - C}{C} = (g - i)\,a_{\overline{n}|i} \rightarrow \text{Let } m = \frac{P - C}{C}$$
$$\frac{m}{a_{\overline{n}|i}} = g - i$$
$$i = g - \frac{m}{a_{\overline{n}|i}}$$

If know g, m then derive $i \Rightarrow$ Bond yield rate

SIMPLE ITERATION

1. Initial guess for $i \Rightarrow$ use formula
2. Second result, use formula
3. If lucky, converges to one value

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Newton Raphson Iteration

If $f(x)=0$ and $f'(x)$ not near zero

let $x_n = x_{n-1} - \dfrac{f(x_{n-1})}{f'(x_{n-1})}$

Guaranteed to converge to right answer

BUT

Not so easy to set up $f(x_{n-1})$ and $f'(x_{n-1})$

BONDS – YIELD RATES

I. Iteration

II. Test answer ranges

III. Use calculator – READ the manual!

CALLABLE BONDS

$P = C + (Fr - Ci)\, a_{\overline{n}|i}$

If bond sells at a premium, assume <u>earliest</u> possible redemption date. This gives <u>lowest</u> price that gives yield rate of i.

$P = C + C(g - i)\, a_{\overline{n}|i} \qquad (i < g)$

If bond is called at a date other than assumed above, the actual yield rate will be higher than what was calculated initially.

NOTE: Price calculation assumes same redemption values at all dates the bond can be called.

CALLABLE BONDS

$P = C + (Fr - Ci)\, a_{\overline{n|}i}$

If bond sells at a discount, assume <u>latest</u> possible redemption date. This gives <u>lowest</u> price that gives yield rate of i.

$P = C + C(g - i)\, a_{\overline{n|}i} \qquad (g < i)$

If bond is called at a date other than assumed above, the actual yield rate will be higher than what was calculated initially.

NOTE: Price calculation assumes same redemption values at all dates the bond can be called.

BONDS - DURATION

Measure of future years implicit in a stream of payments

Simplest idea "method of equated time"

$$\bar{t} = \frac{\sum_{t=1}^{n} t \cdot R_t}{\sum_{t=1}^{n} R_t}$$

t = year

R_t = payment in year

BUT - Missing PV concept!

BONDS - DURATION

Measure of future years implicit in a stream of payments

Duration uses PV of each payment to calculate weighted average:

$$\bar{d} = \frac{\sum_{t=1}^{n} t \cdot v^t \cdot R_t}{\sum_{t=1}^{n} v^t R_t}$$

t = year

R_t = payment in year

duration

$\equiv$ regular duration

$\equiv$ Macaulay duration

BONDS - DURATION

Duration $\bar{d} = \dfrac{\sum_{t=1}^{n} t \cdot v^t \cdot R_t}{\sum_{t=1}^{n} v^t R_t}$

Modified Duration $= \dfrac{\bar{d}}{1+i}$

BONDS - DURATION

10 year bond, annual 8% coupons

$$\bar{d} = \frac{\sum t v^t R_t}{\sum v^t R_t}$$

$$= \frac{1 \cdot v(.08) + 2v^2(.08) + \ldots + 10v^{10}(.08) + 10v^{10}(1)}{v(.08) + v^2(.08) + \ldots + v^{10}(.08) + v^{10}(1)}$$

$$= \frac{.08(Ia_{\overline{10|}}) + 10v^{10}}{.08\, a_{\overline{10|}} + v^{10}}$$

$$\frac{\bar{d}}{1+i} = \text{modified duration}$$

BONDS - DURATION

10 year mortage with level payments

$$\bar{d} = \frac{\sum t v^t R_t}{\sum v^t R_t}$$

$$= \frac{\text{Payment}\left[1 \cdot v + 2 \cdot v^2 + \ldots + 10 v^{10}\right]}{\text{Payment}\left[v + v^2 + \ldots + v^{10}\right]}$$

$$= \frac{Ia_{\overline{10}|}}{a_{\overline{10}|}}$$

$$\frac{\bar{d}}{(1+i)} = \text{modified duration}$$

KEY CONCEPTS

SECTION V - BONDS

1. Amortized value = price

2. Price formulas:
   a. Standard FRANK
   b. Alternative - callable
   c. Makeham - serial

3. Bond price
   a. Between coupon dates
   b. Successive coupon dates

4. Duration / modified duration