SECTION 4

LOANS

AMORTIZATIONS - LOANS

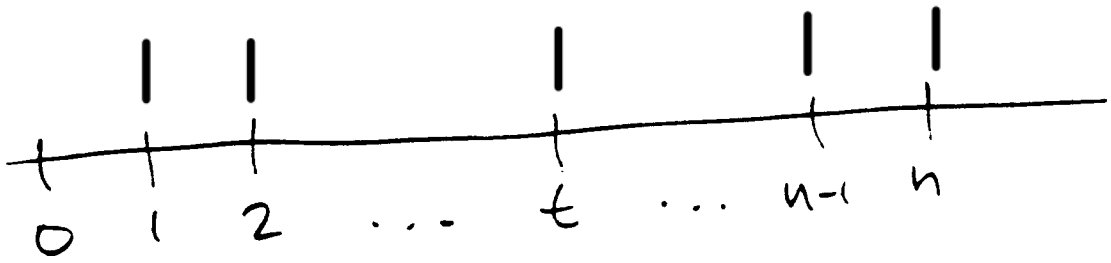
Amount of loan = PV of series of payments valued at time zero. After nth payment, O/S balance of the loan is zero.

At any point in time, O/S balance of loan can be calculated two ways:

Prospective:
O/S loan Balance = PV future payments

Retrospective:
O/S loan Balance = Accum value of loan - accum past payments

OUTSTANDING LOAN BALANCE



At time t, O/s loan balance is

Prospective : $a_{\overline{n-t}|}$

Retrospective : $a_{\overline{n}|}(1+i)^t - s_{\overline{t}|}$

$$= \left[\frac{1-v^n}{i}\right](1+i)^t - \frac{(1+i)^t-1}{i}$$

Proof they are Equal

$$= \frac{(1+i)^t - v^{n-t} - (1+i)^t + 1}{i}$$

$$= \frac{1-v^{n-t}}{i}$$

$$= a_{\overline{n-t}|}$$

AMORTIZATION SCHEDULE

Payment Num	Payment Amount	Interest Paid	Principal Paid	Outstanding Loan
0				$a_{\overline{n}|i} = \frac{1-v^n}{i}$
1	1	$i\left(\frac{1-v^n}{i}\right)$	v^n	$a_{\overline{n-1}|}$
2	1	$1-v^{n-1}$	v^{n-1}	$a_{\overline{n-2}|}$
⋮	⋮	⋮	⋮	⋮
n-1	1	$1-v^2$	v^2	$a_{\overline{1}|}$
n	1	$1-v^1$	v^1	$a_{\overline{0}|}$
TOTAL	n	$n-a_{\overline{n}|}$	$a_{\overline{n}|}$	

NOTES: 1. Based on simple n year loan, payments of \$1 per year
2. Constant interest rate of i
3. Principal payments increase geometrically each year

REFINANCE LOANS

Single loan $1 per year payments
n years of payments
Orig loan $(a\overline{n}|)(\$1)$

After t year O/S Loan $a\overline{n-t}|i$

Change # payments remaining
interest rate
payment amount

Change remaining payments to 5
New payment = $\dfrac{a\overline{n-t}|i}{a\overline{5}|i}$

Change interest rate to j
New payment = $\dfrac{a\overline{n-t}|i}{a\overline{n-t}|j}$

Change payment to \$2, what is new remaining # of payments?
$2(a\overline{x}|i) = a\overline{n-t}|i$

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