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SPRING 2000 EA-1B EXAM SOLUTIONS (Course P-360U)

Solutions last revised November 21, 2003

See revision history on next page

These solutions have been compared with those produced by other technical actuaries, and they represent my best understanding of the correct way to solve these problems. As usual, it seems easy to get an answer in the correct range as long as you are not actually taking the exam!

This was the last EA-1B exam given. Based on the percentage of students who passed this exam, it was also one of the easier ones. Earlier years' exams may be more representative for cost methods problems.

Revision History:

November 21, 2003	Revised solution for problem 3
September 30, 2003	Corrected annuity symbols: problems 2, 8, 9, 10, 12, 13, 15, 16 and 20
December 16, 2002	Revised solution for problem 3
June 25, 2002	Revised solution for problem 17
June 24, 2002	Revised solutions for problems 15 and 18
August 1, 2001	Original solutions

Problem 1

The key to this problem is calculating the normal cost under the Projected Unit Credit (PUC) method. Under PUC, the normal cost is defined based on the change in the "funding accrued benefit" (FAB). The Unit Credit method simply uses the actual accrued benefit.

The 1.412(c)(3)-1 regulations define "funding accrued benefit":

1. Project pay to retirement age
2. Calculate the projected benefit
3. Pro-rate the projected benefit based on service today versus service at retirement. This pro-rata calculation must reflect each year's rate of benefit accrual.

For a final average pay plan, you get the same value for the FAB if you apply the benefit formula to past service, but use projected earnings. For a career average pay plan, you must do the calculation as described in the regulations.

Age 60 at 01/01/00
Age 59 pay 35,000
Age 64 pay 46,838 = $35{,}000\ (1.06)^5$
FAE 3 at 65 44,237 = $46{,}838\ (\ddot{a}_{\overline{3}|.06} / 3)$

Since the benefit accrues at the rate of 2% for each year of service, the change in the FAB is 2% times the FAE at 65. The normal cost is the present value of the change in FAB:

Δ FAB = 2% (FAE 3 at 65)
= 884.73

PUC NC = PV (Δ FAB)
= $884.73\ (D_{65} / D_{60})\ \ddot{a}_{65}^{(12)}$
= $884.73\ (1.08)^{-5}\ (8.33)$

With no pre-retirement decrements, the D / D term is interest only.

PUC NC = 884.73 (.6806) (8.33)
= 5,016

Answer is C

Problem 2 **Revised 09/30/03**

The key to this problem is calculating the normal cost under the Entry Age Normal method. In general, the Entry Age Normal Cost (EANC) is defined as the present value of benefits at entry age, divided by a temporary annuity at entry age. If the EANC is calculated as a level percentage of salary, the temporary annuity will include a salary scale:

Level $ EANC: $PVB_{EA} / \ddot{a}_{\overline{EA:RA-EA}|}$ level at all ages

Level % EANC: $PVB_{EA} / {}^{S}\ddot{a}_{\overline{EA:RA-EA}|}$ at entry age - adjust later ages by salary scale

At entry age, the EANC as a level dollar amount is always greater than the EANC as a level percentage of pay. This is not true for later ages, since the level percentage of pay normal cost is an increasing dollar amount each year.

Participant Smith is age 55 at 01/01/00, which is also the entry age. Based on the prior discussion, the answer is either A, B, or C. With retirement assumed at age 58, each temporary annuity is for three years. Based on the information given, the ratio X / Y equals the ratio of the annuities:

$$X / Y = \ddot{a}_{\overline{EA:RA-EA}|} / {}^{S}\ddot{a}_{\overline{EA:RA-EA}|} = \ddot{a}_{\overline{55:3}|} / {}^{S}\ddot{a}_{\overline{55:3}|}$$

$$\ddot{a}_{\overline{55:3}|\,i} = (N_{55} - N_{58}) / D_{55} = (D_{55} + D_{56} + D_{57}) / D_{55}$$

$${}^{S}\ddot{a}_{\overline{55:3}|\,i} = ({}^{S}N_{55} - {}^{S}N_{58}) / {}^{S}D_{55} = ({}^{S}D_{55} + {}^{S}D_{56} + {}^{S}D_{57}) / {}^{S}D_{55}$$

$${}^{S}D_{X} = (1+s)^{X} D_{X} \text{ by definition}$$

Now plug in the 4% salary scale and the commutation values:

$${}^{S}\ddot{a}_{\overline{55:3}|\,i} = [(1.04)^{55}D_{55} + (1.04)^{56}D_{56} + (1.04)^{57}D_{57}] / [(1.04)^{55}D_{55}]$$

$$= [(1.04)^{0}D_{55} + (1.04)^{1}D_{56} + (1.04)^{2}D_{57}] / [(1.04)^{0}D_{55}]$$

$$X / Y = [(D_{55} + D_{56} + D_{57}) / D_{55}] / [(D_{55} + (1.04)^{1}D_{56} + (1.04)^{2}D_{57}) / D_{55}]$$

$$= [16{,}393 + 15{,}709 + 15{,}035] / [16{,}393 + (1.04)^{1}(15{,}709) + (1.04)^{2}(15{,}035)]$$

$$= 47{,}137 / 48{,}992$$

$$= .9621$$

Answer is B

Problem 3 **Revised 11/21/03**

The key to this problem is handling the multiple retirement decrements correctly in calculating the accrued liability. The Unit Credit accrued liability is defined as the present value of the actual accrued benefit.

Using the old assumptions, this is simply a deferred annuity calculation:

Age 40 at 01/01/00
Service is 10 years
Accrued benefit 2,400 = 10(12)(20)

$$\begin{aligned} \text{UC AL} &= \text{PV of AB} \\ &= 2{,}400\,(D_{61} / D_{40})\,\ddot{a}^{(12)}_{61} \\ &= 2{,}400\,(1.07)^{-21}\,(8.333) \\ &= 4{,}830 \end{aligned}$$

With no pre-retirement decrements, the D / D term is interest only. With retirement decrements, the accrued liability must be calculated as a complicated summation. Since the same benefit is provided at each retirement age after 61, the accrued liability should go down when the decrements are introduced.

$$\text{UC AL} = \sum_{t=0}^{23} v^t \, {}_tp^{(T)}_{40} \, q^{(r)}_{40+t} \, ERB_{40+t} \, \ddot{a}^{(12)}_{40+t}$$

		(1)	(2)	(3)		(4)	(5)	(6)
t	40+t	v^t	${}_tp^{(T)}_{40}$	$q^{(r)}_{40+t}$	$p^{(T)}_{40+t}$	ERB_{40+t}	$\ddot{a}^{(12)}_{40+t}$	(1)(2)(3)(4)(5)
21	61	0.2415	1.00	0.40	0.60	2,400	8.333	1,932
22	62	0.2257	0.60	0.60	0.40	2,400	8.167	1,593
23	63	0.2109	0.24	1.00	0.00	2,400	8.000	972
								4,497

The reduction in the accrued liability is 4,830 – 4,497 = 333.

Answer is D

This problem would be a little harder if the early retirement benefits were reduced before normal retirement age. For a much harder problem that also requires calculation of temporary annuities with multiple retirement decrements, see 1999 #09 and 1994 #15.

Problem 4

The key to this problem is knowing the gain / loss formulas. The total gain /loss is defined as the difference between the expected and actual unfunded accrued liability. The non-investment gain / loss is defined as the difference between the expected and actual accrued liability. The investment gain / loss is defined as the difference between the expected and actual actuarial value of assets.

The problem asks for a mortality loss, which is a non-investment G/L:
Non-inv G/L $= {}_eAL_1 - AL_1$
${}_eAL_1 = (1+i)(AL_0 + NC_0) - (\text{actual benefit payments} + i)$

For an active employee, the benefit payments in the ${}_eAL_1$ formula are zero. For a non-active employee, the normal cost in the ${}_eAL_1$ formula is zero.

Smith retired at 01/01/97 at age 61 with a three year certain and life annuity, with annual payments of 4,800. At 01/01/99, Smith had a one year certain and life annuity at age 63. At 01/01/00 Smith had a life only annuity at age 64.

The two dates 01/01/99 and 01/01/00 correspond to AL_0 and AL_1 :

$$AL_0 = 4{,}800\ \ddot{a}_{\overline{63:1}|}$$
$$= 4{,}800\ (1 + N_{64} / D_{63})$$
$$= 4{,}800\ (1 + N_{64} / [N_{63} - N_{64}])$$
$$= 4{,}800\ \{1 + (31{,}010 / [35{,}623 - 31{,}010])\}$$
$$= 4{,}800\ (7.7223)$$
$$= 37{,}067$$

$$AL_1 = 4{,}800\ \ddot{a}_{64}$$
$$= 4{,}800\ (N_{64} / D_{64})$$
$$= 4{,}800\ (N_{64} / [N_{64} - N_{65}])$$
$$= 4{,}800\ (31{,}010 / [31{,}010 - 26{,}867])$$
$$= 35{,}928$$

$${}_eAL_1 = (1+i)(AL_0 + \text{zero}) - (\text{actual benefit payments} + i)$$
$$= 1.07(37{,}067) - 1.07(4{,}800)$$
$$= 34{,}526$$

Mortality loss = 1,402 = 35,928 – 34,526

Answer is B

Problem 5

The key to this problem is that the retirement gain / loss calculation is simply the difference between two accrued liability values, one as an active employee, and one as a retired employee. Another point is that you must calculate the monthly life annuities at both early retirement and assumed retirement age.

In general, under the Unit Credit method, you would expect a loss upon early retirement. The Unit Credit accrued liability is defined as the present value of the actual accrued benefit. As an active employee, this is simply a deferred annuity calculation:

Age 60 at 01/01/00
Service is 20 years
Accrued benefit 4,800 = 20(12)(20)

Active AL = PV of AB
= $4{,}800\ (D_{65} / D_{60})\ \ddot{a}^{(12)}_{65}$
= $4{,}800\ (N^{(12)}_{65} / D_{60})$
= $4{,}800\ (N_{65} - (11/24)D_{65}) / D_{60}$
= 4,800 (904 - (11/24)99) / 151
= 4,800 (5.6863)
= 27,294

Retired AL = PV of Early retirement benefit (ERB)
ERB = 4,800 * (1-5(5%))
= 3,600

Retired AL = $3{,}600\ \ddot{a}^{(12)}_{60}$
= $3{,}600\ (N^{(12)}_{60} / D_{60})$
= $3{,}600\ (N_{60} / D_{60} - 11/24)$
= 3,600 (1,547 / 151 - 11/24)
= 3,600 (9.7867)
= 35,232

The accrued liability as a retiree is greater, so there is a loss upon retirement at 01/01/00. The loss is the difference of 35,232 - 27,294 = 7,938.

Answer is A

Problem 6 - Page 1

The key to this problem is carefully handling the change in interest rate. You should calculate the 01/99 valuation at 7% interest, and the 01/00 valuation at 8% interest. The Unit Credit accrued liability is defined as the present value of the actual accrued benefit. The Unit Credit normal cost is calculated as the present value of the change in the accrued benefit.

Valuation date	**1/1/99**	**1/1/00**
Age	59	60
Service	21	22
Interest rate	7%	8%
Accrued Benefit	21(12)(25) = 6,300	22(12)(25) = 6,600
Accrued Liability	$6{,}300(1.07)^{-6}(9.70)$ = 40,720	$6{,}600(1.08)^{-5}(8.74)$ = 39,259
Normal Cost	AL / 21 (Shortcut calculation) = 1,939	

Once the valuation results are done, there are only a few calculations to arrive at the answer. The questions asks for the Unfunded accrued liability, which is calculated as the excess of the accrued liability over the actuarial value of assets:

01/00 UAL = AL - AAV

To calculate this UAL, you need to determine the assets at 01/99, and bring them forward to 01/00 based on the 10% actual return during 1999:

01/99 UAL = AL - AAV
AAV = AL - UAL
= 40,720 - 12,000
= 28,720

01/00 AAV = 1.10 (28,720 + 1,939)
= 33,725
UAL = AL - AAV
= 39,259 - 33,725
= 5,534

Answer is C

Problem 6 - Page 2

Another method for working this problem is to attack it as a gain / loss problem. This is not the best method of solution, since it is slightly longer. The question asks for the 01/00 UAL, which can be defined in a slightly different manner:

01/00 UAL = ${}_eUAL$ - asset gain - assumption change
01/00 ${}_eUAL$ = (1+i)($NC_0 + UAL_0$) - (contrib. + interest)
= 1.07(NC_0 + 12,000) - (1.07)(NC_0)
= 12,840

The asset gain can't be calculated without the amount of the 01/99 normal cost. You'll also need to do 01/00 valuation results for the assumption change impact on the UAL.

Valuation date	**1/1/99**	**1/1/00**
Age	59	60
Service	21	22
Interest rate	7%	8%
Accrued Benefit	21(12)(25)	22(12)(25)
	= 6,300	= 6,600
Accrued Liability	$6,300(1.07)^{-6}(9.70)$	$6,600(1.08)^{-5}(8.74)$
	= 40,720	= 39,259
Normal Cost	AL / 21	(Shortcut calculation)
	= 1,939	

The asset gain is the difference between the 7% valuation rate and the actual return of 10%, applied to the 01/99 asset value and the 1999 contribution:
01/99 AAV = AL - UAL
= 40,720 - 12,000
= 28,720

AAV gain = (10% - 7%)(28,720 + 1,939)
= 920

The assumption change base is the difference in the AL at 01/00 on both 7% and 8%:
01/00 7% AL = ${}_eAL_1$ (since there are no pre-retirement decrements)
= 45,645 = 1.07(40,720 + 1,939)
Assump ΔAL = 6,386 = 45,645 - 39,259

01/00 UAL = ${}_eUAL$ - asset gain - assumption change
= 12,840 - 920 - 6,386
= 5,534

Answer is C

Problem 7 - Page 1

The key to this problem is that the Unit Credit normal cost is not level each year. The problem asks for the unfunded accrued liability (UAL) at 01/00. Based on what you are given, you can calculate the Accrued Liability and the Actuarial value of assets at 01/00, which will give you the UAL.

The Unit Credit normal cost is calculated as the present value of the change in the accrued benefit. With no pre-retirement decrements, the normal cost will grow with interest each year. In addition, the Accrued Liability can be brought forward each year based on the formula for the expected accrued liability:

01/98 NC = 4,000 (given)
01/99 NC = 4,320 = 1.08 * 4,000

01/99 AL = ${}_eAL_1$ (no pre-retirement decrements)
01/99 ${}_eAL_1$ = 1.08 (50,000 + 4,000)
= 58,320

01/00 AL = ${}_eAL_1$ (no pre-retirement decrements)
01/00 ${}_eAL_1$ = 1.08 (58,320 + 4,320)
= 67,651

Now you can bring forward the asset values. For 1998, the assets earned 8%, but for 1999, you are told that the assets earned 9.5%

01/99 AAV = 1.04 (6,000) (using simple interest for ½ year)
= 6,240
01/00 AAV = 1.095 (6,240 + 6,200)
= 13,622

01/00 UAL = AL - AAV
= 67,651 - 13,622
= 54,029

Answer is E

If you used compound interest for the 1998 year, you will get a different numerical answer, but one in the same range:
01/99 AAV = $(1.08)^{½}$ (6,000) (using compound interest for ½ year)
= 6,235
01/00 AAV = 1.095 (6,235 + 6,200)
= 13,617

01/00 UAL = AL - AAV
= 67,651 - 13,617
= 54,034

Problem 7 - Page 2

Another method for working this problem is to attack it as a gain / loss problem. This is slightly less obvious than the previous solution given. The question asks for the 01/00 UAL, which can be defined in a slightly different manner:

01/98 NC = 4,000 (given)
01/99 NC = 4,320 = 1.08 * 4,000

01/99 UAL = ${}_eUAL$ (no gains or losses)
01/99 ${}_eUAL = (1+i)(NC_0 + UAL_0)$ - (contrib. + interest)
= 1.08(4,000 + 50,000) - 1.04(6,000) (using simple interest)
= 52,080

01/00 UAL = ${}_eUAL$ - asset gain
01/00 ${}_eUAL$ = 1.08(4,320 + 52,080) - 1.08(6,200)
= 54,216

The asset gain is the difference between the 8% valuation rate and the actual return of 9.5%, applied to the 01/99 asset value and the 1999 contribution:

01/99 AAV = 1.04(6,000) (from above - using simple interest)
= 6,240

AAV gain = (9.5% - 8%)(6,240 + 6,200)
= 187

01/00 UAL = ${}_eUAL$ - asset gain
= 54,216 - 187
= 54,029

Answer is E

If you used compound interest for the 1998 year, you will get a different numerical answer, but one in the same range. Just be careful to use the same interest crediting technique in the calculation of both the AAV and the expected UAL.

Problem 8 **Revised 09/30/03**

The key to this problem is handling the change in marital status, and calculating the normal cost under the Entry Age Normal method. In general, the Entry Age Normal Cost (EANC) is defined as the present value of benefits at entry age, divided by a temporary annuity at entry age.

The effect of the change in marital status is to increase the present value of benefits. Instead of a life annuity factor, the PVB at 65 will be based on a joint and 100% last survivor annuity factor.

Level $ EANC: $PVB_{EA} / \ddot{a}_{\overline{EA:RA-EA}|}$ (assume level $ unless pay related benefit)

Age 64 at 01/01/00 Spouse is also age 64
Entry age 40
Total service 25
Projected benefit 50,000 (given)

PVB_{EA} = PV of Projected benefit
= $50{,}000\ (D_{65} / D_{40})\ \ddot{a}_{65}^{(12)}$
= $50{,}000\ (1.07)^{-25}\ (8.74)$
= 80,517

EA NC = $80{,}517 / \ddot{a}_{\overline{40:25}|}$
= $80{,}517 / \ddot{a}_{\overline{25}|.07}$ (no pre-retirement decrements)
= 6,457

Once the participant becomes married, the annuity at 65 changes to a joint and 100% last survivor annuity factor:

$\ddot{a}_{\frac{(12)}{65:65}} = \ddot{a}_{65}^{(12)} + 100\% \ (\ddot{a}_{65}^{(12)} - \ddot{a}_{65:65}^{(12)})$
$= 8.74 + (8.74 - 6.90)$
$= 10.58$

New NC = $6{,}457 * \ddot{a}_{\frac{(12)}{65:65}} / \ddot{a}_{65}^{(12)}$
= 7,817
Δ EANC = 1,359

Answer is A

Past problems similar to this one were slightly harder due to unusual formula reductions to the benefit for married participants.

Problem 9 **Revised 09/30/03**

The key to this problem is handling the salary scale correctly. In addition, you should recognize that this is similar to a side fund problem. You need to calculate the total projected benefit at assumed retirement age, and deduct the paid up annuity.

Then you can apply the Individual Aggregate (IA) cost method to the net benefit. In general, the IA present value of normal costs (PVNC) is defined as the present value of benefits less the allocated assets.

Age 50 at 01/01/00
Age 49 pay 50,000
Age 64 pay 83,767 = $50{,}000\ (1.035)^{15}$
FAE 3 at 65 80,967 = $83{,}767\ (\ddot{a}_{\overline{3}|.035} / 3\ )$

Proj ben = 50%(FAE 3 at 65)
= 40,483
Net ben = 40,483 – 12(1,250)
= 25,483

PVB = $25{,}483\ (\ D_{65} / D_{50}\ )\ \ddot{a}^{(12)}_{65}$
= $25{,}483\ (1.07)^{-15}\ (8.736)$ (no pre-retirement decrements)
= 80,689

PVNC = 80,689 – 7,500
= 73,189

The IA normal cost is calculated by dividing the PVNC by a temporary annuity from current age to the assumed retirement age. Since the problem asks for the level dollar normal cost, there is no salary scale in the temporary annuity.

IA NC = $73{,}189 / \ddot{a}_{\overline{50:15}|}$
= $73{,}189 / \ddot{a}_{\overline{15}|\ .07}$ (no pre-retirement decrements)
= 7,510

Answer is C

Problem 10 - Page 1 **Revised 09/30/03**

The key to this problem is knowing the gain / loss formulas, and how to calculate the normal cost under the Entry Age Normal method. In general, the Entry Age Normal Cost (EANC) is defined as the present value of benefits at entry age, divided by a temporary annuity at entry age.

One way to work the problem is to solve for the total amount of gain / loss. The total G/L is defined as the difference between the expected and actual UAL values.

01/00 G/L = ${}_eUAL_1 - UAL_1$
${}_eUAL_1$ = $(1+i)(NC_0 + UAL_0)$ - (contrib. + interest)
UAL_1 = AL_1 - 4,234

Now you need to calculate the values for the EANC, AL_0, and AL_1:
Level $ EANC: $PVB_{EA} / \ddot{a}_{\overline{EA:RA-EA}|}$

Age 40 at 01/01/99
Entry age 35
Total service 30
Projected benefit 9,000 = 30(25)(12)

PVB_{EA} = $9{,}000 (D_{65} / D_{35}) \ddot{a}^{(12)}_{65}$
= 9,000 (94,414 / 894,190)(8.736)
= 9,000 (.1056)(8.736)
= 8,302

$\ddot{a}_{\overline{35:30}|}$ = $(N_{35} - N_{65}) / D_{35}$
= (12,364,661 - 868,053) / 894,190
= 12.8570
EANC = $8{,}302 / \ddot{a}_{\overline{35:30}|}$
= 645.69

Now you can calculate both EAN accrued liabilities using the retrospective definition:

	01/01/99	**01/01/00**
Retrospective EAN AL	EANC ($\ddot{s}_{\overline{35:5}|}$)	EANC ($\ddot{s}_{\overline{35:6}|}$)
$\ddot{s}_{\overline{35:N}|}$	$(N_{35} - N_{40}) / D_{40}$ = (12,364,661 - 8,452,737)/ 632,275 = 6.1871	$(N_{35} - N_{41}) / D_{41}$ = (12,364,661 - 7,820,462)/ 589,655 = 7.7065
Retrospective EAN AL	3,995 = 645.69 * 6.1871	4,976 = 645.69 * 7.7065

Problem 10 - Page 2

$_eUAL_1$ = $1.07(NC_0 + [AL_0 - 3{,}000]) - 934$
= $1.07 (645.69 + 995) - 934$
= 821

UAL_1 = 4,976 - 4,234
= 742

01/00 G/L = $_eUAL_1 - UAL_1$
= 821 - 742
= Gain of 79

Answer is C

Another method of solution calculates the each component of G/L separately. The non-investment gain / loss is defined as the difference between the expected and actual accrued liability. The investment gain / loss is defined as the difference between the expected and actual actuarial value of assets.

The investment G/L is a short calculation:

$_eAAV_1$ = $1.07(3{,}000) + 934$
= 4,144
AAV_1 = 4,234
Gain = 90.00

Non-inv G/L = $_eAL_1 – AL_1$
$_eAL_1$ = $(1+i)(AL_0 + NC_0)$ – (actual benefit payments + i)

To save some arithmetic, we can convert this into an equivalent formula. This allows us to calculate only one accrued liability to determine the G/L:

$_eAL_1$ = $(1+i)(AL_0 + NC_0)$ – (zero for active)
= $(1+i)(p_x / p_x)(AL_0 + NC_0)$ (x is last year's age)
= $p_x [(1+i) / p_x](AL_0 + NC_0)$
= $p_x [D_x / D_{x+1}](AL_0 + NC_0)$
= $p_x * AL_1$

Problem 10 - Page 3 **Revised 09/30/03**

This last formula is based on retrospective definition of Accrued Liability, which is the retrospective accumulation of all prior normal costs. For an active employee who survives, the mortality loss is $AL_1 - {}_eAL_1$, which equals $q_x * AL_1$. To use this, you must derive the value of q_x from the values of D_x:

$(1+i) / p_x = D_x / D_{x+1}$

p_{40} $= (1.07) * D_{41} / D_{40}$
$= 1.07 (589,655 / 632,275)$
$= .9979$

q_{40} $= 1 - .9979$
$= .0021$

The next step is calculation of the accrued liability at age 41. This can be done using a different formula, which does not require you to separately calculate the EANC:

EAN AL $= PVB_{CA} * (\ddot{a}_{\overline{EA:CA-EA}|} / \ddot{a}_{\overline{EA:RA-EA}|})$ (for level $ EANC)

PVB_{CA} $= 9,000 (D_{65} / D_{41}) \ddot{a}^{(12)}_{65}$
$= 9,000 (94,414 / 589,655)(8.736)$
$= 12,589$

EAN AL $= 12,589 [N_{35} - N_{41}] / [N_{35} - N_{65}]$
$= 12,589 [12,364,661 - 7,820,462] / [12,364,661 - 868,053]$
$= 12,589 [4,544,199 / 11,496,608]$
$= 4,976$

Mort. loss $= 4,976 * .0021$
$= 10.58$

Net Gain = Asset gain - mortality loss
= 90.00 - 10.58
= 79.42

Answer is C

Problem 11

The key to this problem is calculating the normal cost under the Entry Age Normal method. In general, the Entry Age Normal Cost (EANC) is defined as the present value of benefits at entry age, divided by a temporary annuity at entry age. Since the plan benefits are based on pay, the EANC is calculated as a level percentage of salary, and the temporary annuity will include a salary scale.

In this problem, you are given the old plan NC at entry age. You need to calculate the EAN accrued liability under both the old plan and the new plan. The old plan AL is calculated as the retrospective accumulation of the old plan NC, allowing for the fact that the dollar amount increases at each age.

Age 55 at 01/01/00
Age 50 at entry

NC for age …… 50 51 … 54

"Old" AL = $10{,}000\ [\ (1.07)^5 + (1.07)^4(1.05)^1 + \ldots + (1.07)^1(1.05)^4\ ]$

= $10{,}000\ (1.07)^5\ [\ 1 + (1.05/1.07)^1 + \ldots + (1.05/1.07)^4\ ]$

= $10{,}000\ (1.07)^5\ \ddot{a}_{\overline{5}|j}$ where 1+j = 1.07 / 1.05, and j = 1.90%

= 10,000 (1.4026)(4.8165)

= 67,555

The EAN AL and EANC are calculated using PVB at entry age, which is based on the projected benefit. The new plan AL can be calculated by multiplying the old plan AL by the ratio of the new plan benefit to the old plan benefit.

Age 54 pay P (assumed)
Age 64 pay $P\ (1.05)^{10}$
FAE 3 at 65 $P\ (1.05)^{10}\ (\ddot{a}_{\overline{3}|.05} / 3\ )$

Old Ben = 50%(FAE 3 at 65)
= $.50(P\ )(1.05)^{10}\ (\ddot{a}_{\overline{3}|.05} / 3\ )$

New Ben = 60%(Pay at 64)
= $.60(P\ )(1.05)^{10}$
ratio = $.60 / (.50\ \ddot{a}_{\overline{3}|.05} / 3\ )$

= .60 / (.50*.9531)

New AL = [.60 / (.50*.9531)] 67,555
= 85,051
AL chg = 17,497

Answer is C

Problem 12 - Page 1 **Revised 09/30/03**

The key to this problem is carefully handling the salary scale, and calculating the normal cost under the Aggregate method. Under the Aggregate method, the present value of normal costs (PVNC) is defined as the present value of benefits less the assets. The Aggregate normal cost is calculated by dividing the PVNC by a temporary annuity from current age to the assumed retirement age.

In this problem, the plan benefit is based on pay, so the temporary annuity will include the salary scale. If all assumptions are met, the resulting normal cost should be level as a percentage of pay each year. The following solution shows all the calculations at 01/00 and 01/01. As you'll see later, there are some shortcuts for the calculations at 01/01.

Valuation date	**1/1/2000**	**1/1/2001**
Age	45	46
Prior age pay	50,000	1.07(50,000) = 53,500
Age 64 pay	$50{,}000(1.05)^{20}$	$53{,}500(1.05)^{19}$
	= 132,665	= 135,192
Projected benefit	60% (132,665)	60% (135,192)
	= 79,599	= 81,115
PV future benefits	$79{,}599\ (D_{65} / D_{45})\ \ddot{a}^{(12)}_{65}$	$81{,}115\ (D_{65} / D_{46})\ \ddot{a}^{(12)}_{65}$
	$79{,}599\ (1.07)^{-20}\ (8.736)$	$81{,}115\ (1.07)^{-19}\ (8.736)$
	= 179,698	= 195,940
Actuarial Assets	0	1.10(10,685) *
	0	= 11,754
PVNC	179,698 - 0	195,940 - 11,754
	= 179,698	=184,186
Normal cost	$179{,}698 / {}^{S}\ddot{a}_{\overline{45:20}\rceil}$	$184{,}186 / {}^{S}\ddot{a}_{\overline{46:19}\rceil}$

* Note: the asset value at 01/01 is calculated based on the final 01/00 normal cost, whose calculation is shown on the next page.

Problem 12 - Page 2 **Revised 09/30/03**

As was shown in 2000 #11, you can evaluate the temporary annuity with salary scale by using an annuity at a different interest rate:

$${}^{S}\ddot{a}_{\overline{45:20}|} = 1 + (1.05/1.07)^{1} + \ldots + (1.05/1.07)^{19} \quad \text{(20 terms for NC at ages 45 to 64)}$$

$$= \ddot{a}_{\overline{20}|\,j} \qquad \text{where } 1+j = 1.07 / 1.05, \text{ and } j = 1.90\%$$

Valuation date	**1/1/2000**	**1/1/2001**
Normal cost	$179{,}698 / {}^{S}\ddot{a}_{\overline{45:20}|}$	$184{,}186 / {}^{S}\ddot{a}_{\overline{46:19}|}$
	$= 179{,}698 / \ddot{a}_{\overline{20}|\,1.90}$	$= 184{,}186 / \ddot{a}_{\overline{19}|\,1.90}$
	$= 179{,}698 / 16.8171$	$= 184{,}186 / 16.1183$
	$= 10{,}685$	$= 11{,}427$

Answer is D

There is an alternative method of solution, which is to treat this as more of a gain / loss problem. This is slightly less obvious than the previous solution given.

The idea is that, if all assumptions are met, the resulting normal cost should be level as a percentage of pay each year. You still have to do all the valuation calculations at 01/00, but the calculations at 01/01 are done a little bit quicker than in the prior solution.

Valuation date	**1/1/2000**
Age	45
Prior age pay	50,000
Age 64 pay	$50{,}000(1.05)^{20}$
	$= 132{,}665$
Projected benefit	$60\% (132{,}665)$
	$= 79{,}599$
PV future benefits	$79{,}599 (D_{65} / D_{45}) \ddot{a}^{(12)}_{65}$
	$79{,}599 (1.07)^{-20} (8.736)$
	$= 179{,}698$
Actuarial Assets	0
PVNC	179,698

Problem 12 - Page 3 **Revised 09/30/03**

Valuation date **1/1/2000**

Normal cost

$$179{,}698 / {}^{S}\ddot{a}_{\overline{45:20}|}$$

$$= 179{,}698 / \ddot{a}_{\overline{20}|1.90}$$

$$= 10{,}685$$

Now you should set up a short table to show what the expected and actual values are at 01/01, based on the information given. The assets earned more than expected, and the salary increase was higher than expected.

Scenario	**1/1/2001 Expected**	**1/1/2001 Actual**
PV future benefits	1.07(179,698)	1.07(179,698)(1.07/1.05) = 195,940
Actuarial Assets	1.07(0+10,685)	1.10(0+10,685) = 11,754
PVNC	(don't care)	195,940 - 11,754 = 184,186
PVE / E	$16.1183 = \ddot{a}_{\overline{19}|1.90}$	$16.1183 = \ddot{a}_{\overline{19}|1.90}$
Normal cost	(don't care)	184,186 / 16.1183 = 11,427

The 01/01 normal cost is exactly the same result calculated earlier. If you did calculate the normal cost in the expected column, it will be last year's value with one year's increase at the salary scale, or 1.05(10,685).

Problem 13 **Revised 09/30/03**

The key to this problem is calculating the accrued liability under the Entry Age Normal method. In addition, you need to use the information under the Unit Credit method to derive values under Entry Age Normal. You also must know how to handle the change in plan benefits.

Age 54 at 01/01/99
Past service 19
Future service 8 (to assumed retirement age 62)
Total service 27

You are given the Unit Credit accrued liability at 01/99, which is the present value of the accrued benefit. You can calculate the total PV of future benefits directly from that value. Eventually you can use the total PVB at 01/00 to calculate the EAN accrued liability.

Accrued benefit 19(12)(20)
Projected benefit 27(12)(20)

Active AL = PV of Accrued Benefit
= 24,910 = 19(12)(20) $(D_{62} / D_{54})\ \ddot{a}^{(12)}_{62}$

Total PVB = PV of Projected benefit
= (27/19) (24,910)
= 35,398

At 01/00, the total PVB will have grown by one year's interest, due to no decrements:
01/00 PVB = 37,876 = 1.07(35,398)

The next step is calculation of the EAN accrued liability. This can be done using a formula that does not require you to separately calculate the EANC:

EAN AL = $PVB_{CA} * (\ddot{a}_{\overline{EA:CA-EA}|} / \ddot{a}_{\overline{EA:RA-EA}|})$ (for level \$ EANC)
= 37,876 * $(\ddot{a}_{\overline{35:20}|} / \ddot{a}_{\overline{35:27}|})$
= 37,876 * $(\ddot{a}_{\overline{20}|.07} / \ddot{a}_{\overline{27}|.07})$ (no pre-retirement decrements)
= 33,476

New plan AL = (25/20) 33,476
Change in AL = (5/20) 33,476
= 8,369

Answer is D

You could have solved for $\ddot{a}^{(12)}_{62}$ instead of using ratios to determine the total PVB.

Problem 14

The key to this problem is handling the multiple retirement decrements correctly in calculating the accrued liability. This problem is similar to 2000 #3, but with the added complication of early retirement reduction factors.

Age 59 at 01/01/00
Service is 20 years
Accrued benefit 4,800 = 20(12)(20)

The Unit Credit accrued liability is defined as the present value of the actual accrued benefit. With retirement decrements, the accrued liability must be calculated as a complicated summation.

$$\text{UC AL} = \sum_{t=0}^{6} v^t \, {}_tp_{59}^{(T)} \, q_{59+t}^{(r)} \, ERB_{40+t} \, \ddot{a}_{59+t}^{(12)}$$

		(1)	(2)	(3)		(4)	(5)	(6)
t	59+t	v^t	${}_tp_{59}^{(T)}$	$q_{59+t}^{(r)}$	$p_{59+t}^{(T)}$	ERB_{59+t}	$\ddot{a}_{59+t}^{(12)}$	(1)(2)(3)(4)(5)
1	60	0.9346	1.000	0.50	0.50	3360	10.248	16,090
2	61	N/A	0.500	-	1.00	N/A	N/A	-
3	62	0.8163	0.500	0.75	0.25	3936	9.849	11,867
4	63	N/A	0.125	-	1.00	N/A	N/A	-
5	64	N/A	0.125	-	1.00	N/A	N/A	-
6	65	0.6663	0.125	1.00	0.00	4800	9.206	3,681
								31,638

As shown in the calculations above, you don't need to waste time calculating values for ages with no retirement decrements. The early retirement benefits are calculated by applying the 6% per year reductions to the accrued benefit of 4,800:

ERB_{62} = 4,800 (1-.06(65-62))
ERB_{60} = 4,800 (1-.06(65-60))

Answer is B

Problem 15 **Revised 09/30/03**

The key to this problem is calculating the normal cost under the Individual Level Premium method. In general, the Individual Level Premium (ILP) Normal Cost is defined as the sum of multiple layers. A new layer is established each time the plan benefit changes, and it funds the change in the present value of future benefits prospectively over future service:

Δ ILP NC = PV (Δ Proj Benefit) / $\ddot{a}_{\overline{X:RA-X}|}$ level \$ normal cost

The point of the problem is that the projected benefit will change each time that pay changes. Since there are three pay values, there are three separate layers of normal cost. With no salary scale, the problem is not as messy as some past ILP problems.

Hire Age 48 at 01/01/98
Future Service 17

Normal cost calculation date

	01/01/98	01/01/99	01/01/00
Age	48	49	50
Annual Pay	50,000	45,000	55,000
Projected benefit	25,000	22,500	27,500
Δ Projected benefit	25,000 *	-2,500	5,000
PV (Δ Proj Benefit)	$25{,}000v^{17}\ddot{a}^{(12)}_{65}$	$-2{,}500v^{16}\ddot{a}^{(12)}_{65}$	$5{,}000v^{15}\ddot{a}^{(12)}_{65}$
Δ Normal cost	$25{,}000\,\ddot{a}^{(12)}_{65} / \ddot{s}_{\overline{17}|.07}$	$-2{,}500\,\ddot{a}^{(12)}_{65} / \ddot{s}_{\overline{16}|.07}$	$5{,}000\,\ddot{a}^{(12)}_{65} / \ddot{s}_{\overline{15}|.07}$
Annuity certain value	32.9990	29.8402	26.8881
Δ Normal cost	6,618	-732	1,625
Total normal cost	6,618	5,886	7,511

* NOTE: Some students don't like this identification of the initial normal cost layer. I consider that their benefit increases from zero to 25,000 when they enter the plan.

Answer is B

Problem 16 **Revised 09/30/03**

The key to this problem is calculating the accrued liability under the Entry Age Normal method. This can be done using a formula that does not require you to separately calculate the EANC:

$$\text{EAN AL} = PVB_{CA} * \left(\ddot{a}_{\overline{EA:CA-EA}|} \Big/ \ddot{a}_{\overline{EA:RA-EA}|} \right)$$

There is nothing mysterious or hidden about this problem. You need to calculate the projected benefit, present value of future benefits, and the EAN accrued liability.

$$\text{EAN AL} = PVB_{CA} * ([N_{EA} - N_{CA}] / D_{EA}) / ([N_{EA} - N_{RA}] / D_{EA})$$
$$= PVB_{CA} * [N_{EA} - N_{CA}] / [N_{EA} - N_{RA}]$$

$$PVB_{CA} = \text{(Projected benefit)} (D_{RA} / D_{CA}) \ddot{a}^{(12)}_{RA}$$

	Smith	**Brown**
Age	40	58
Entry age	35	40
Total service	30	25
Projected benefit	50%(12)(2,500) = 15,000	50%(12)(3,500) = 21,000
PV future benefits	$15{,}000 (D_{65} / D_{40}) \ddot{a}^{(12)}_{65}$ = 15,000 (97/651)(8.748) = 19,552	$21{,}000 (D_{65} / D_{58}) \ddot{a}^{(12)}_{65}$ = 21,000 (97/174)(8.748) = 102,412
Accrued liability	$19{,}552 [N_{35} - N_{40}] / [N_{35} - N_{65}]$ 19,552 (12,727 - 8,701)/(12,727 - 893) = 19,552 * .3402 = 6,652	$102{,}412 [N_{40} - N_{58}] / [N_{40} - N_{65}]$ 102,412 (8,701 - 1,862)/(8,701 - 893) = 102,412 * .8759 = 89,702

The total accrued liability is 96,354.

Answer is A

This problem could be much messier if it had a non-zero salary scale, such as 2000 #20.

Problem 17 - Page 1 **Revised 06/25/02**

The key to this problem is handling the multiple retirement decrements correctly in calculating the normal cost under the Projected Unit Credit (PUC) method. Under PUC, the normal cost is defined based on the change in the "Funding accrued benefit" (FAB). The Unit Credit method simply uses the actual accrued benefit. For a final average pay plan, you can calculate the FAB based on past service and the benefit accrual formula, but with final average pay projected to the benefit commencement age.

Age 45 at 01/01/00
Past service 10 years

The PUC normal cost is defined as the present value of the change in the funding accrued benefit. With retirement decrements, the normal cost must be calculated as a complicated summation.

$$\text{PUC NC} = \sum_{t=0}^{20} v^t \, {}_tp_{45}^{(T)} \, q_{45+t}^{(r)} \, \Delta FAB_{45+t} \, \ddot{a}_{45+t}^{(12)}$$

The problem asks for a single term of the summation that corresponds to retirement at age 56. One key point is that you need to project pay to age 55, which would be used in the calculation of the age 56 benefit.

Age 44 pay 50,000
Age 55 pay 76,973 = $50{,}000\ (1.04)^{11}$
FAE 1 at 56 76,973

Smith was hired at age 35, and has 10 years of service. The change in the FAB is 2.0% times the FAE at 56. Note that you should not use the benefit accrual based on service at age 56. You should calculate the change in the Funded accrued benefit, which is based on past service.

Δ FAB = 2%(FAE at 56)(ERF at 56)
= 2%(76,973)(.73)
= 1,123.80

Problem 17 - Page 2 **Revised 06/25/02**

The normal cost is the present value of the change in FAB. This is complicated by the multiple retirement decrements. The single term for age 56 is calculated as follows:

$$v^t \quad {}_tp_{45}^{(T)} \quad q_{45+t}^{(r)} \quad \Delta FAB_{45+t} \quad \ddot{a}_{45+t}^{(12)} \qquad \text{where } t=11$$
$$= (1.07)^{-11} \; (1\text{-}.25) \; (.10) \; (1{,}123.80) \; (12.4)$$
$$= 496.54$$

Answer is D

For more complicated multiple retirement decrements, see 2000 #14

NOTE:

If you look carefully at the prior expression, the annuity is identified as $\ddot{a}_{45+t}^{(12)}$, which is a monthly annuity. The problem gave the value of $\ddot{a}_{56}$ as 12.40, which is an annual annuity, instead of the monthly annuity used above.

It appears the problem was defective, because you must assume the annuity value is monthly to get the official answer of D. If you use the given annual annuity value of 12.40, and adjust it to a monthly annuity, the resulting answer range is C.

$$\ddot{a}_{56} = 12.40$$
$$\ddot{a}_{56}^{(12)} = \ddot{a}_{56} - 11/24$$
$$= 11.942$$

The single term for age 56 is calculated as follows:

$$v^t \quad {}_tp_{45}^{(T)} \quad q_{45+t}^{(r)} \quad \Delta FAB_{45+t} \quad \ddot{a}_{45+t}^{(12)} \qquad \text{where } t=11$$
$$= (1.07)^{-11} \; (1\text{-}.25) \; (.10) \; (1{,}123.80) \; (11.942)$$
$$= 478.18$$

Answer is C

Problem 18 **Revised 06/24/02**

The key to this problem is knowing the gain / loss formulas. In addition, you must allow for the fact that there is still a liability at 01/01/00 after Smith's death. Unlike 1998 #13, you can NOT ignore the period certain annuity payments. The reason is that there is a different assumption basis used for the certain annuity after Smith's death.

Non-inv G/L $= {}_eAL_1 - AL_1$
${}_eAL_1 = (1+i)(AL_0 + NC_0) - (\text{actual benefit payments} + i)$

Now calculate the annuities for the AL_0 and AL_1 values. I prefer to use monthly interest rates to calculate the monthly certain annuities:

5% per annum is equivalent to .407% per month $(1.05)^{1/12} = 1.00407$
7% per annum is equivalent to .565% per month $(1.07)^{1/12} = 1.00565$

Date	01/01/95	01/01/99	01/01/00
Age	65	69	N/A
Payment form	15 years C&L	11 years C&L	10 years certain
		$= \ddot{a}^{(12)}_{\overline{69:11}\rceil}$	$= \ddot{a}^{(12)}_{\overline{10}\rceil.05}$
		$= \ddot{a}^{(12)}_{\overline{11}\rceil.07} + (D_{80} / D_{69})\ \ddot{a}^{(12)}_{80}$	$= \ddot{a}^{(12)}_{\overline{10}\rceil.05}$
		$= (\ddot{a}_{\overline{132}\rceil.00565}/12) + N^{(12)}_{80} / D_{69}$	$= \ddot{a}_{\overline{120}\rceil.00407}/12$
		= 93.36 / 12 + (91,357/64,805)	= 95.15 / 12
		= 9.1898	= 7.9293

$AL_1 = 12{,}000 * 7.9293$
$= 95{,}152$

${}_eAL_1 = (1+i)(AL_0 + \text{zero}) - (\text{actual benefit payments} + i)$
$= 1.07(12{,}000)(9.1898 - \ddot{a}^{(12)}_{\overline{1}\rceil.07})$
$= 1.07(12{,}000)[9.1898 - (\ddot{a}_{\overline{12}\rceil.00565}/12)]$
$= 1.07(12{,}000)[9.1898 - .9696]$
$= 12{,}000\ (8.7956)$
$= 105{,}547$

Mortality gain = 10,395 = 105,547 - 95,152.

Answer is A

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Problem 19

The key to this problem is knowing how to calculate the actuarial equivalent late retirement benefit. In general, actuarially equivalent benefits have the same present value. For an annuity form with no death benefit, the actuarial equivalent factor is the ratio of the $N_x^{(12)}$ values:

$$(\text{Normal benefit}_{65})\ \ddot{a}_{65}^{(12)} = (\text{Late benefit}_x)\ (D_X / D_{65})\ \ddot{a}_X^{(12)}$$

$$(\text{Normal benefit}_{65})\ N_{65}^{(12)} = (\text{Late benefit}_x)\ N_X^{(12)}$$

$$\text{Late benefit}_x = (\text{Normal benefit}_{65})\ N_{65}^{(12)} / N_X^{(12)}$$

The rest of the problem consists of calculating the plan benefit amounts at both retirement dates, as well as the actuarially increased benefit.

Date	01/01/99	01/01/00
Age	65	66
Retirement type	Normal	Late
Service	30	31
3 year FAE	57,667	59,000
Accrued benefit	2%(30)(57,667) = 34,600	2%(31)(59,000) = 36,580

The actuarially increased benefit at age 66 is based on the normal retirement benefit:

$$\text{Late benefit}_{66} = (\text{Normal benefit}_{65})\ N_{65}^{(12)} / N_{66}^{(12)}$$
$$= 34{,}600\ (\ 3{,}230{,}371 / 2{,}918{,}698\ )$$
$$= 38{,}295$$

Smith receives the greater of the plan benefit at 66, or the actuarially increased benefit. The resulting benefit is 38,295.

Answer is E

Problem 20 - Page 1

The key to this problem is knowing the gain / loss formulas, and how to calculate the accrued liability under the Entry Age Normal method. In addition, you must carefully handle the salary scale, which complicates this problem quite a bit.

There are two methods of solution for this problem. The brute force technique involves calculation of the EANC, and then the accrued liability, and the mortality loss. This is left as an exercise for the student.

The preferred method mirrors what was done as the alternative solution for 2000 #10. This does not require calculation of the EANC, and eliminates some of the arithmetic. The first step is calculation of the projected benefit:

Age 61 at 01/01/01
Age 60 pay 50,000
Age 64 pay 60,775 = $50{,}000\ (1.05)^4$
FAE 3 at 65 57,927 = $60{,}775\ (\ddot{a}_{\overline{3}|.05} / 3\ )$

Proj ben = 2%(30)(57,927)
= 34,756

The non-investment gain / loss is defined as the difference between the expected and actual accrued liability.

Non-inv G/L = ${}_eAL_1 - AL_1$
${}_eAL_1$ = (1+i)($AL_0 + NC_0$) – (actual benefit payments + i)

To save some arithmetic, we can convert this into an equivalent formula. This allows us to calculate only one accrued liability to determine the G/L:

${}_eAL_1$ = (1+i)($AL_0 + NC_0$) – (zero for active)
= (1+i)(p_x / p_x)($AL_0 + NC_0$) (x is last year's age)
= p_x [(1+i) / p_x]($AL_0 + NC_0$)
= p_x [D_x / D_{x+1}]($AL_0 + NC_0$)
= p_x * AL_1

This last formula is based on retrospective definition of Accrued Liability, which is the retrospective accumulation of all prior normal costs. For an active employee who survives, the mortality loss is $AL_1 - {}_eAL_1$, which equals $q_x * AL_1$. To use this, you must derive the value of q_x from the values of D_x.

Problem 20 - Page 2 **Revised 09/30/03**

$(1+i) / p_x = D_x / D_{x+1}$

p_{60} $= (1.07) * D_{61} / D_{60}$

$= 1.07 (133,046 / 144,405)$

$= .985833$

q_{60} $= 1 - .985833$

$= .014167$

The next step is calculation of the accrued liability at age 61. This can be done using a different formula, which does not require you to separately calculate the EANC. This is similar to the formula from 2000 #10, except that this level % of pay EANC is determined using annuities with a salary scale:

$$\text{EAN AL} = PVB_{CA} * ({}^{s}\ddot{a}_{\overline{EA:CA-EA}|} / {}^{s}\ddot{a}_{\overline{EA:RA-EA}|}) \qquad \text{(for level \% EANC)}$$

PVB_{CA} $= 34,756 (D_{65} / D_{61}) \ddot{a}^{(12)}_{65}$

$= 9,000 (94,414 / 133,046)(8.7358)$

$= 215,462$

EAN AL $= 215,462 [{}^{S}N_{35} - {}^{S}N_{61}] / [{}^{S}N_{35} - {}^{S}N_{65}]$

$= 215,462 [138,500,016 - 39,917,788] / [138,500,016 - 30,013,858]$

$= 215,462 [98,582,228 / 108,486,158]$

$= 195,792$

Mort. loss $= 195,792 * .014167$

$= 2,774$

Answer is D

As mentioned earlier, it takes a while longer if you first calculate the EANC at age 35. Then you need to adjust it with the salary scale to age 61 to calculate the retrospective accrued liability.

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