SoftwarePolish

Rick Groszkiewicz
2964 Nestle Creek Drive
Marietta, GA 30062-4857

Voice/fax (770) 971-8913

email: rickg@softwarepolish.com

SPRING 1999 EA-1B
EXAM SOLUTIONS

Revision History:	
06/24/02	Revised solutions for problems 1 and 9

Spring 1999 EA-1B

It may be my imagination, but this exam seemed much harder than recent EA-1B exams. There were more problems with salary scales, and more long problems. I believe it was extremely difficult to even work all 20 problems within the allotted time, much less get enough correct to pass the exam.

Spring 1999 EA-1B

1 The Aggregate method is an aggregate type cost method. One unique characteristic of the Aggregate method is that the normal cost will be level as a percentage of pay if (i) all assumptions are met, and (ii) the contribution plus interest is equal to the normal cost plus interest.

To determine the 1999 normal cost, you must develop the value of the assets at 1-1-99. You are told nothing about the prior contributions. The key to working the problem is that you must assume the prior contributions at 1-1 were equal to the normal cost. There is no other way to work this problem.

The first step is calculation of the 1-1-96 normal cost based on 20,000 earnings. This same value of normal cost would be used for 1997 and 1998 as well, since there is no salary scale, and all assumptions were met each year.

1-1-96 age 35 past service -0- total service 30

Agg NC = (PVB - AAV) / (average $\ddot{a}_{x:\overline{RA-x}|}$)

Projected Ben = 50% (20,000) = 10,000

1-1-96 PVB = $10{,}000\ \ddot{a}_{65}^{(12)}\ D_{65}/D_{35}$

$NC = 10{,}000\,(10.5)(1.06)^{-30} / \ddot{a}_{\overline{30}|.06}$ no pre-ret decrement

$= 105{,}000 / \ddot{s}_{\overline{30}|.06}$

$= 1{,}252.96$

Spring 1999 EA-1B

(1) The quickest way to work the problem is to calculate the change in the normal cost at 1-1-99. The only reason the normal cost increases is due to the increase in salary scale.

Δ Pay $= 20{,}000$

Δ Proj Ben $= 10{,}000$

1-1-99 Δ PVB $= 10{,}000(10.5)(1.06)^{-27}$

Δ NC $= \dfrac{105{,}000}{\ddot{s}_{\overline{27|}.06}}$

$= 1{,}554.91$

1-1-99 total NC $= 2{,}807.87 = 1554.91 + 1252.96$

C

The "longer" way to work the problem is to develop the assets at 1-1-99, and then calculate the NC:

1-1-97 asset value $= (1.06)(1996\text{ NC}) = \ddot{s}_{\overline{1|}.06}(1252.96)$

1-1-99 asset value $= \ddot{s}_{\overline{3|}.06}(1252.96) = 4{,}228$

1-1-99 Projected Ben $= 20{,}000 = (.50)(40{,}000)$

PVB $= 20{,}000\,(10.50)(1.06)^{-27}$

$= 43{,}547$

PVNC $= 39{,}319 = 43{,}547 - 4{,}228$

NC $= 39{,}319 / \ddot{a}_{38:\overline{27|}}$

$= 39{,}319 / \ddot{a}_{\overline{27|}.06}$

$= 2{,}807.87$

Spring 1999 EA-1B

2 The key to working this problem is that the normal cost will be level as a percentage of pay, if all assumptions are met. Since you are given the 1-1-99 normal cost, the 1-1-98 normal cost must be $2{,}080/1.04 = 2{,}000$.

You can use the relationship for either the expected accrued liability, or the expected unfunded. It is slightly quicker to calculate expected unfunded at 1-1-99:

$$\text{1-1-99 } {}_eUAL = (1+i)(NC_0 + UAL_0) - (\text{contrib} + \text{interest})$$
$$= 1.07(2{,}000 + 114{,}400 - 91{,}500) - 1.07(3{,}000)$$
$$= 1.07(24{,}900 - 3{,}000)$$
$$= 23{,}433$$

Since all assumptions were met, this is also equal to the actual value of the 1-1-99 UAL:

$$\text{1-1-99 } UAL = 23{,}433 = AL - AAV$$
$$AL = UAL + AAV = 23{,}433 + 95{,}800$$
$$= 119{,}233$$

A

If you use the formula for the expected accrued liability, you have to solve for the actual benefits paid:

$$\text{1-1-99 } eAL = (1+i)(NC_0 + AL_0) - (\text{actual benefits paid} + \text{interest})$$
$$= 1.07(2{,}000 + 114{,}400) - (BP+I)$$
$$\text{1-1-99 } eAAV = (1+i)(AAV_0) + (\text{contrib} + \text{interest}) - (\text{benefits} + \text{interest})$$
$$= 1.07(91{,}500 + 3{,}000) - (BP+I)$$
$$95{,}800 = 101{,}115 - (BP+I) \Rightarrow BP+I = 5{,}315$$
$$\therefore \text{1-1-99 } AL = eAL = 1.07(116{,}400) - 5{,}315 = 119{,}233$$

Spring 1999 EA-1B

3 This is a fairly typical mortality gain/loss problem. The key to working the problem is knowing how to calculate the experience G/L. You also must derive an annuity value based on the factors you are given.

The general formula for the non-investment G/L is

non-inv G/L = $eAL_1 - AL_1$

$eAL_1 = (1+i)(NC_0 + AL_0)$ - (actual benefit pmts + interest)

With retired participants, the NC_0 term is zero. Based on the ages of Smith and Brown at 1-1-98 and 1-1-99, you must derive values of $\ddot{a}_{66}$ and $\ddot{a}_{67}$ to calculate AL_0 and AL_1, and finally, the 1998 mortality loss:

$1 + v p_x \ddot{a}_{x+1} = \ddot{a}_x \Rightarrow \ddot{a}_{x+1} = (\ddot{a}_x - 1.0)(1+i)/p_x$

$\ddot{a}_{66} = (\ddot{a}_{65} - 1.0)(1+i)/p_{65}$

$\ddot{a}_{67} = (\ddot{a}_{66} - 1.0)(1+i)/p_{66}$

$a_{65:\overline{1}|} = v p_{65} = .9135$

$p_{65} = .9135(1.07)$

$\ddot{a}_{66} = (9.194 - 1.0)(1.07)/[.9135(1.07)]$

$= 8.194/.9135$

$= 8.9699$

$a_{65:\overline{2}|} = v p_{65}(1 + v p_{66})$

$1.7460 = .9135(1 + v p_{66})$

$1.9113 = 1 + v p_{66}$

$p_{66} = .9113(1.07)$

Spring 1999 EA-1B

(3) $\ddot{a}_{67} = (\ddot{a}_{66} - 1.0)(1+i)/p_{66}$

$= (8.9699 - 1.0)(1.07)/[.9113(1.07)]$

$= 7.9699/.9113$

$= 8.7453$

Finally, you have values for both $\ddot{a}_{66}$ and $\ddot{a}_{67}$!

	Smith	Brown
Birth date	1-1-33	1-1-32
1-1-98 Age	65	66
eAL_1	$1.07(12{,}000)\ddot{a}_{65}$	$1.07(15{,}000)\ddot{a}_{66}$
	$-1.07(12{,}000)$	$-1.07(15{,}000)$
	$= 1.07(12{,}000)(8.194)$	$= 1.07(15{,}000)(7.9699)$
	$= 105{,}211$	$= 127{,}917$
AL_1	$12{,}000\,\ddot{a}_{66}$	$15{,}000\,\ddot{a}_{67}$
	$= 12{,}000(8.9699)$	$= 15{,}000(8.7453)$
	$= 107{,}639$	$= 131{,}180$
Loss	2,428	3,263 $\Sigma = 5{,}691$
		C

Spring 1999 EA-1B

4 The key to this problem is carefully handling the change in assumed retirement age. This will affect the future service, benefit amount, age reduction factor on the benefit, annuity at retirement, and discounting period.

Birth date 1-1-40 Hire date 1-1-65
1-1-99 Age 59
Past service 34 Benefit past service 25
(limited by plan formula)
Accrued benefit 12($25)25 = 7,500

	Assumed Retirement Age	
	62	65
Early retirement factor	1-3(.025)	1.00
Future service	3	6
PV factor at 1-1-99	$v^3 \ddot{a}_{62}^{(12)}$	$v^6 \ddot{a}_{65}^{(12)}$
	$=(1.07)^{-3}$ 10.750	$=(1.07)^{-6}(9.815)$
	= 8.7752	= 6.5401
UCAL = PV accd benefit	7500(.925)8.7752	7500(1.0)(6.5401)
	= 60,878	= 49,051

Δ = 11,827

B

Spring 1999 EA-1B

5 FIL is an aggregate type cost method. You are given last year's valuation data so you can calculate the expected unfunded liability this year. Since you have the present value of benefits, and $\ddot{a}_{62:\overline{3}|}$, it looks like you can calculate the normal cost fairly quickly.

Unfortunately, it is not that simple. The key to working this problem is that you are given the present value of <u>retirement</u> benefits. You must determine the present value of death benefits and include that in the PVNC. If you don't, the resulting normal cost of 8,058 is in the wrong answer range.

The present value of the death benefit is

$$100{,}000\, A^1_{62:\overline{3}|} = 100{,}000\left(v\,q_{62} + v^2\left({}_{1|}q_{62}\right) + v^3\left({}_{2|}q_{62}\right)\right)$$

You have to express this in terms of the annuity values that you are given:

$$A^1_{62:\overline{3}|} = A_{62} - \frac{D_{65}}{D_{62}} A_{65}$$

$${}_3E_{62} = \frac{D_{65}}{D_{62}} \qquad A_x = 1 - d\ddot{a}_x$$

$$A^1_{62:\overline{3}|} = (1 - d\ddot{a}_{62}) - \frac{D_{65}}{D_{62}}(1 - d\ddot{a}_{65})$$

$$= 1 - \frac{D_{65}}{D_{62}} - d\left(\ddot{a}_{62} - \frac{D_{65}}{D_{62}}\ddot{a}_{65}\right)$$

Spring 1999 EA-1B

(5) $A'_{62:\overline{3}|} = 1 - {}_3E_{62} - d(\ddot{a}_{62:\overline{3}|})$

$= 1 - .77127 - \frac{.07}{1.07}(2.7612)$

$= .04809$

$\therefore$ PV of death benefit $= 4{,}809$

Total PVB $= 50{,}000 + 4{,}809 = 54{,}809$

FIL Normal cost $= \dfrac{PVNC}{\text{average } \ddot{a}_{x:\overline{RA-x}|}}$

$PVNC = PVB - AAV - UAL$

FIL UAL = eUAL by definition

$= (1+i)(NC_0 + UAL_0) - (\text{Contrib} + \text{interest})$

$= 1.07(5{,}000 + 20{,}000) - 4{,}000$

$= 22{,}750$

FIL PVNC $= 54{,}809 - 5{,}000 - 22{,}750$

$= 27{,}059$

$\ddot{a}_{x:\overline{RA-x}|} = \ddot{a}_{62:\overline{3}|} = 2.7612$

FIL NC $= 9{,}800$

D

Spring 1999 EA-1B

6 This problem was defective, and was eliminated from the scoring of the exam. The reason is that the first column labeled ${}^{s}N_x$ should be labeled ${}^{s}D_x$.

The wording in the question is unusual; these problems normally ask for the accrued liability, which can be calculated on a retrospective basis as the accumulation of past normal costs. In this problem, that is not the preferred approach. It will be faster to use the commutation functions.

The Entry Age Normal accrued liability can be calculated three different ways:

Retrospective $\quad (EANC_{40})\ {}^{s}\ddot{s}_{35:\overline{5|}}$

Prospective $\quad PVFB - (EANC_{40})({}^{s}\ddot{a}_{40:\overline{25|}})$

Alternative $\quad PVFB\left(\dfrac{{}^{s}\ddot{a}_{35:\overline{5|}}}{{}^{s}\ddot{a}_{35:\overline{30|}}}\right)$

In most problems, the third approach will be fastest when you are given salary scale based commutation functions

$$\text{1-1-99 PVFB} = (\text{Proj Benefit})\ \ddot{a}_{65}^{(12)}\ \frac{D_{65}}{D_{40}}$$

Birth date 1-1-59 $\quad$ Hire date 1-1-94

1-1-99 Age 40 $\quad$ Age 40 pay = 30,000

Past svc 5 $\quad$ Age 64 pay = $30{,}000(1.04)^{24}$

Total Svc 30 $\quad$ = 76,899

Spring 1999 EA-1B

(6) FAE57 at 65 $= 76{,}899(\ddot{a}_{\overline{5}|7.04}/5)$

$= 71{,}207$

Proj Benefit $= 2\%(30)71{,}207$

$= 42{,}724$

1-1-99 PVFB $= 42{,}724(8.7358)(94{,}414/632{,}275)$

$= 55{,}732$

1-1-99 EAN AL $= 55{,}732\left(\dfrac{{}^{S}N_{35} - {}^{S}N_{40}}{{}^{S}N_{35} - {}^{S}N_{65}}\right)$ denominators cancel out

$= 55{,}732\left(\dfrac{83{,}833{,}438 - 67{,}201{,}791}{83{,}833{,}438 - 14{,}869{,}249}\right)$

$= 13{,}441$

(D)

I'll show the retrospective definition, which is longer due to the need for calculation of the EANC:

1-1-94 EANC $= (PVB_{35} / {}^{S}\ddot{a}_{35:\overline{30}|})$

PVB $= 42{,}724\, \ddot{a}_{65}^{(12)} D_{65}/D_{35}$

$= 39{,}408$

${}^{S}\ddot{a}_{35:\overline{30}|} = ({}^{S}N_{35} - {}^{S}N_{65})/{}^{S}D_{35}$

$= 19.5446$

$EANC_{35} = 2{,}016$

1-1-99 $EANC_{40} = (EANC_{35})(1.04)^5$ Level as % of pay

$= 2{,}453$

EAN AL $= EANC_{40}\,({}^{S}\ddot{s}_{35:\overline{5}|})$

$= 2{,}453\,({}^{S}N_{35} - {}^{S}N_{40})/{}^{S}D_{40}$

$= 13{,}441$

It only looks shorter because most steps, details were skipped

Spring 1999 EA-1B

7 The key to working this problem is interpreting what you are asked to calculate (X%). In effect, you are calculating a level % of pay normal cost to fund the difference in the benefits. No cost method is stated, but it is similar to the Aggregate or the Individual Aggregate definition of the normal cost.

1-1-99 Age 50   Age 50 pay = 50,000
Past svc 15   Age 64 pay = 98,997 = $50{,}000(1.05)^{14}$
Total svc 30

Old plan benefit $25(2\%)(98{,}997)$
New plan benefit $30(1.5\%)(98{,}997)$
Δ plan benefit $5\%(98{,}997) = 4{,}950$

$$\Delta NC = \frac{4950\, \ddot{a}_{65}^{(12)}\, D_{65}/D_{50}}{{}^{s}\ddot{a}_{50:\overline{15}|}}$$

$${}^{s}\ddot{a}_{50:\overline{15}|} = 1 + v\,p_{50}(1.05) + (1.07)^{-2}\,{}_2p_{50}(1.05)^2 + \ldots + (1.07)^{-14}\,{}_{14}p_{50}(1.05)^{14}$$
$$= 1 + (1.05/1.07) + \ldots + (1.05/1.07)^{14}$$ no pre-ret decrements
$$= \ddot{a}_{\overline{15}|j} \text{ where } 1+j = 1.07/1.05 = 1.0190$$

$$\Delta NC = \frac{4950(8.74)(1.07)^{-15}}{13.1878}$$
$$= 1{,}189$$
$\Delta NC\% = 2.38\% = 1{,}189/50{,}000$ A

Spring 1999 EA-1B

8 This is a typical Individual Level Premium problem. Under ILP, each change in benefit produces a new layer of normal cost. You are given the 1-1-99 normal cost as 500, which is the sum of two layers, corresponding to the original plan at 1-1-95, and the new plan at 1-1-99.

You should set up two columns to determine each normal cost layer. Then you can solve for the value of K

	1-1-95 Plan	1-1-99 Plan
Age at plan change	40	44
Total service	25	25
Projected benefit	12(\$10)25 = 3,000	12(\$K)25 = 300K
Δ Projected Ben	3,000	300K − 3000
Δ ILP NC	$\dfrac{3{,}000\, \ddot{a}_{65}^{(12)} D_{65}/D_{40}}{\ddot{a}_{40:\overline{25}\rvert}}$	$\dfrac{(300K-3000)\, \ddot{a}_{65}^{(12)} D_{65}/D_{44}}{\ddot{a}_{44:\overline{21}\rvert}}$

Under the ILP method, the change in PVB is funded over future service (most often as level dollar normal cost).

Δ ILP NC	$\dfrac{3000(8.53)}{\ddot{s}_{\overline{25}\rvert .07}}$	$\dfrac{(300K-3000)(8.53)}{\ddot{s}_{\overline{21}\rvert .07}}$

$$\text{Total NC} = 500 = 378.1226 + (53.3061K - 533.061)$$
$$= 53.3061K - 154.9384$$
$$K = 654.9384/53.3061$$
$$= 12.29$$

C

Spring 1999 EA-1B

9 This is very similar to 1994 #15. This is an extremely difficult problem, since you must use the multiple retirement decrements to calculate both the present value of benefits, and the temporary annuity for the normal cost. The main key to working this problem is handling the temporary annuity correctly.

1-1-99 Age 62 $PVB = \sum_{t=1}^{4} v^t \, {}_tp_{62} \, q^{(r)}_{62+t} \, \ddot{a}^{(12)}_{62+t} \, (RetBen)_{62+t}$

Minor twist is assumption of retirements beyond normal retirement age (handled in the benefit definition "on or after").

Projected Service t	Age $62+t$	(1) $q^{(r)}_{62+t}$	(2) ${}_tp^{(T)}_{62}$	$p^{(T)}_{62+t}$	(3) v^t	(4) $\ddot{a}^{(12)}_{62+t}$	(5) $(RetBen)_{62+t}$	(1)(2)(3)(4)(5) PVB_{62+t}
1	63	.40	1.00	.60	$(1.07)^{-1}$	8.96	20(12)(1)(.90)	723.50
2	64	.60	.60	.40	$(1.07)^{-2}$	8.74	20(12)(2)(.95)	1253.17
3	65	.80	.24	.20	$(1.07)^{-3}$	8.51	20(12)(3)(1.00)	960.31
4	66	1.00	.048	—	$(1.07)^{-4}$	8.29	20(12)(4)(1.00)	291.43
								3228.41

The Entry Age normal cost is defined as $PVB_{EA} / \ddot{a}_{EA:\overline{RA-EA}|}$, and the previous calculation is $PVB_{EA} = PVB_{62}$.

$$\ddot{a}_{62:\overline{4}|} = 1 + v\,p^{(T)}_{62} + v^2\,{}_2p^{(T)}_{62} + v^3\,{}_3p^{(T)}_{62} + v^4\,{}_4p^{(T)}_{62}$$

$$= 1 + \frac{.60}{1.07} + \frac{.24}{(1.07)^2} + \frac{.048}{(1.07)^3} = 1.8096$$

EANC = 3228.41 / 1.8096 = 1784.09 (B)

Other formulas for $\ddot{a}_{62:\overline{4}|}$ are: $\ddot{a}_{\overline{4}|} - (1-.60)v^1 - (1-.24)v^2 - (1-.048)v^3$

or: $\ddot{a}_{\overline{1}|}(.40) + \ddot{a}_{\overline{2}|}(.36) + \ddot{a}_{\overline{3}|}(.192) + \ddot{a}_{\overline{4}|}(.048)$

THIS PAGE WAS INTENTIONALLY LEFT BLANK

Spring 1999 EA-1B

10 This is a typical Projected Unit Credit problem. Under PUC, the definition of the normal cost and accrued liability are similar to Unit Credit:

PUC AL = PV of FAB PUC NC = PV of ΔFAB

The FAB is the funding accrued benefit. This should be calculated by applying the benefit formula based on past service to the projected final average earnings at retirement. In this problem, part of the benefit is not based on pay, and part is.

Birth date 1-1-39 1-1-99 Age 60 Age 60 pay 35,000
Hire date 1-1-79 Past svc 20 Age 64 pay $= 35{,}000(1.04)^4$

$= 40{,}945$

Age 65 $FAE_{\overline{3}|} = \dfrac{40{,}945\, \ddot{a}_{\overline{3}|.04}}{3}$

$= 39{,}390$

1-1-99 PUC AL = PV(FAB)

$FAB = 12(\$10)(20) + 1\%(39{,}390)20$

$= 2{,}400 + 7{,}878$

$= 10{,}278$

$PUC\ AL = 10{,}278\, \ddot{a}_{65}^{(12)}\, D_{65}/D_{60}$

$= 10{,}278(8.7358)\dfrac{94{,}414}{144{,}405}$

$= 58{,}704$

A

THIS PAGE WAS INTENTIONALLY LEFT BLANK