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11 This is a typical messy Entry Age Normal problem with salary scales. The key to working this problem is carefully allowing for the change in the final average earnings period that is part of the change in benefits at 1-1-99.

There are three way to calculate the EAN Accrued Liability (see 1999 #6). With salary scales and commutation functions given, this is the fastest method:

$$EAN\ AL = PVB\left(\frac{{}^{s}\ddot{a}_{EA:\overline{CA-EA|}}}{{}^{s}\ddot{a}_{EA:\overline{RA-EA|}}}\right)$$

Since the annuity factors are unaffected by the change in benefits, the main part of the problem is calculating the change in benefits, and the change in PVB.

Birth date 1-1-49   1-1-99 Age 50   Age 50 pay 30,000
Hire date 1-1-79   entry age 30   Age 64 pay 51,950 $=30{,}000(1.04)^{14}$
Past svc 20   Total svc 35

	Old Plan-FAE5	New Plan-FAE3
FAE at 65	$51{,}950(\ddot{a}_{\overline{5|}.04}/5)$ $=48{,}105$	$51{,}950(\ddot{a}_{\overline{3|}.04}/3)$ $=49{,}978$
Projected Benefit	$.01(35)(48{,}105)$	$.01(35)(49{,}978)$

$\Delta = .01(35)1872 = 655.53$

$\Delta PVB = 655.53\,\ddot{a}^{(12)}_{65}\,D_{65}/D_{50}$

$= 1740.46 = 655.53(8.7358)(94{,}414/310{,}647)$

$\Delta EAN\ AL = \Delta PVB\left({}^{s}\ddot{a}_{30:\overline{20|}}/{}^{s}\ddot{a}_{30:\overline{35|}}\right)$

$= 1740.46\left(\frac{{}^{s}N_{30}-{}^{s}N_{50}}{{}^{s}N_{30}-{}^{s}N_{65}}\right) = 1740.46\left(\frac{103{,}134{,}571-40{,}718{,}455}{103{,}134{,}571-14{,}861{,}249}\right) = 1230.64$ B

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12 This is a typical mandatory employee contribution problem. There is nothing tricky about it, and it is the easiest problem on this exam!

Under the Aggregate method, the definitions are:

$PVNC = PVB - AAV$

$$NC = \frac{PVNC}{\text{average } \ddot{a}_{x:\overline{RA-x}|}} \quad \text{or} \quad NC = \frac{PVNC}{\text{average } {}^{s}\ddot{a}_{x:\overline{RA-x}|}}$$

With mandatory employee contributions, you treat the present value of future mandatory contributions as an additional asset:

$PVNC = PVB - AAV - PV(EEC)$

Since employee contributions are 2% of compensation, you can "approximate" the present value of employee contributions as 2% (present value of compensation). In most plans, mandatory employee contributions are paid at the end of the year, and this calculation is not quite correct - thus the word "approximate":

$$PV(EEC) = 2\%(5{,}000{,}000) = 100{,}000$$

$$PVNC = PVB - AAV - PV(EEC)$$

$$PVB = 1{,}250{,}000 = 1{,}100{,}000 + 100{,}000 + 50{,}000$$

$$PVNC = 1{,}250{,}000 - 600{,}000 - 100{,}000$$

$$= 550{,}000$$

$$\text{Average } \ddot{a}_{x:\overline{RA-x}|} = PVE/E = 5{,}000{,}000/500{,}000 = 10.0$$

$$NC = 550{,}000/10.0 = 55{,}000$$

(b)

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13 This is a typical Attained Age Normal problem. In the initial year of the plan, the IAL is calculated under the Unit Credit cost method. Then the AAN normal cost is calculated as most Aggregate type methods do:

$$\text{AAN NC} = \frac{PVNC}{\text{average } \ddot{a}_{x:\overline{RA-x}|}} \quad \text{or} \quad \frac{PVNC}{\text{average } {}^{S}\ddot{a}_{x:\overline{RA-x}|}}$$

The key points to this problem are correctly calculating the accrued benefit for Unit Credit, the projected benefit for AAN, and allowing for the different number of participants at each age. You should set up two columns to calculate the necessary items

	Birth Date 1-1-54	Birth Date 1-1-49
Number of participants	5	2
Hire date	1-1-89	1-1-94
1-1-99 Past svc	10	5
Individual accrued ben	12(\$25)(10) = 3,000	12(\$25)(5) = 1,500
1-1-99 Age	45	50
Lifeweighted 1-1-99 Present value factor	$5(\ddot{a}^{(12)}_{65} D_{65}/D_{45})$ $= 5(8.74)(1.07)^{-20}$ $= 11.2929$	$2(\ddot{a}^{(12)}_{65} D_{65}/D_{50})$ $= 2(8.74)(1.07)^{-15}$ $= 6.3356$
UCAL = Present value of accrued ben	11.2929(3,000) = 33,879	6.3356(1,500) = 9,503

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(13) The AAN UAL = IAL − AAV
= UCAL − 0
= 43,382 = 33,879 + 9,503 − 0

Now you need to calculate the PVB, the projected benefit, and $\ddot{a}_{x:\overline{RA-x}|}$. Since the benefits are not based on pay, the annuity is not based on salary scale:

	Birth Date		
	1-1-54	1-1-49	
1-1-99 Age	45	50	
$\ddot{a}_{x:\overline{RA-x}|}$	$\ddot{a}_{45:\overline{20}|}$	$\ddot{a}_{50:\overline{15}|}$	
	$= \ddot{a}_{\overline{20}|.07}$	$= \ddot{a}_{\overline{15}|.07}$	
	= 11.3356	= 9.7455	
Total weighted $\ddot{a}_{x:\overline{RA-x}|}$	5(11.3356)	2(9.7455)	
	= 56.6780	= 19.4909	Σ = 76.1689
Average $\ddot{a}_{x:\overline{RA-x}|}$	76.1689/7 = 10.8813		
1-1-99 Total svc	30 $\xrightarrow{\text{limit}}$ to 25	20	
Individual projected ben	12[25(10) + 30(15)]	12[25(10) + 30(10)]	
	= 8,400	= 6,600	
prior page: Life weighted PV factor	11.2929	6.3356	
1-1-99 PVB	94,860	41,815	Σ = 136,675

PVNC = PVB − AAV − UAL
= 136,675 − 0 − 43,382 = 93,293

NC = PVNC / avg $\ddot{a}_{x:\overline{RA-x}|}$ = 93,293/10.8813
= 8,574

B

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14 This is a complicated problem on the Individual Aggregate cost method. When you calculate the normal cost on a level dollar basis, it will not change as long as all assumptions are met. The normal cost on a level % of pay basis will increase by the salary scale each year.

The point of this problem is that the level % of pay normal cost will increase in the future, and will eventually exceed the level dollar normal cost. You need to calculate the values for both normal costs, and determine when they will "cross over".

Birthdate 1-1-49   1-1-99 Age 50   Age 50 pay 30,000

Hire date 1-1-99   Total service 15   Age 64 pay $30{,}000(1.025)^{14}$

$= 42{,}389$

Age 65 FAE3 $= 42{,}389\left(\dfrac{\ddot{a}_{\overline{3}|2.5\%}}{3}\right)$

$= 41{,}364$

Projected benefit $= .02(41{,}364)15$

$= 12{,}409$

Under the Individual Aggregate cost method, assets are allocated to each participant. Since this is the initial year of the plan, the assets are zero. Then the PVNC is calculated on an individual basis as PVB-AAV.

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(14) The IA normal cost is calculated as

$$\frac{PVNC}{\ddot{a}_{x:\overline{RA-X}|}} \quad \text{or} \quad \frac{PVNC}{{}^{s}\ddot{a}_{x:\overline{RA-X}|}}$$

$$\text{Level \$ NC} = \frac{PVB - AAV}{\ddot{a}_{50:\overline{15}|}}$$

$$= \frac{12{,}409\,\ddot{a}_{65}^{(12)}\,D_{65}/D_{50} - 0}{\ddot{a}_{50:\overline{15}|}}$$

$$= \frac{12{,}409(8.74)}{\ddot{s}_{\overline{15}|.07}} \qquad \text{no pre-ret decrements}$$

$$= 4{,}034$$

$$\text{Level \% NC} = \frac{PVB - AAV}{{}^{s}\ddot{a}_{50:\overline{15}|}}$$

$$= \frac{12{,}409(8.74)(1.07)^{-15}}{\ddot{a}_{50:\overline{15}|j}} \qquad \text{where } 1+j = \frac{1.070}{1.025} = 1.0439$$

For a detailed explanation of the salary weighted annuity calculation, see 1999 #7.

$$\text{Level \% NC} = \frac{12{,}409(8.74)(.3624)}{11.2961} = 3480$$

n years from now, the level % NC will be $3480(1.025)^n$.
The year the values cross over, we have $3480(1.025)^n \geq 4{,}034$

$(1.025)^n \geq 1.159$

$n(\log 1.025) \geq \log 1.159$

$n \geq 5.9798 \qquad \therefore$ 1-1-99 + 6 years = 1-1-05 (B)

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15 This question has appeared on the exam several times recently. The key to working this problem is to derive a present value factor at age 65 which reflects the different annuity values for married and unmarried participants. You can use the ratio of the two values based on the different % assumed married to adjust the PVB for active employees.

% married assumption

	Old - 70%	New - 90%
PV factor at 65	$.30(\ddot{a}_{65}^{(12)}) + .70(1.25)(\ddot{a}_{65}^{(12)})$	$.10(\ddot{a}_{65}^{(12)}) + .90(1.25)(\ddot{a}_{65}^{(12)})$
	$= \ddot{a}_{65}^{(12)}(.3 + .7(1.25))$	$= \ddot{a}_{65}^{(12)}(.1 + .9(1.25))$
	$= \ddot{a}_{65}^{(12)}(1.175)$	$= \ddot{a}_{65}^{(12)}(1.225)$

New/old Ratio = 1.225/1.175

New PVB = (1.225/1.175) 2,000,000

= 2,085,106

Note that the change in assumption of % married does not affect the PVB for the people already retired. The rest of the problem is the normal cost calculation under the Aggregate method. You need to derive the average temporary annuity based on the "old" results:

$$\text{Old NC} = \frac{\text{PVNC}}{\text{average } \ddot{a}_{x:\overline{RA-x}|}} = \frac{2{,}300{,}000 - 1{,}000{,}000}{\text{avg } \ddot{a}_{x:\overline{RA-x}|}} = 90{,}000$$

$$\text{average } \ddot{a}_{x:\overline{RA-x}|} = 1{,}300{,}000/90{,}000 = 14.4444$$

new PVNC = 2,085,106 + 300,000 - 1,000,000 = 1,385,106

new NC = 95,892 = 1,385,106 / 14.4444 B

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16 This is a difficult Projected Unit Credit problem. The PUC normal cost is calculated as the change in the funding accrued benefit (FAB). The FAB is calculated by applying the benefit formula based on past service to the projected final average pay.

The key to working this problem is to write expressions for the FAB under the new plan and the old plan. This makes it easier to calculate the ΔFAB, and also minimizes arithmetic errors. You should set up separate columns for Smith, Brown, and Green:

	Smith	Brown	Green
Birth Date	1-1-49	1-1-51	1-1-39
1-1-99 Age	50	48	60
Hire Date	1-1-74	1-1-76	1-1-94
1-1-99 Service	25	23	5
Current Pay	30,000	40,000	50,000
Age 64 Pay	$30{,}000(1.02)^{14}$	$40{,}000(1.02)^{16}$	$50{,}000(1.02)^{4}$
	= 39,584	= 54,911	= 54,122
1-1-99 Old Plan FAB	.03(20)(39,584)	.03(20)(54,911)	.03(5)(54,122)
1999 Δ Old FAB	zero	zero	.03(54,122)
1-1-99 New Plan FAB	.04(25)(39,584)	.04(23)(54,911)	.04(5)(54,122)
1999 Δ New FAB	zero	.04(54,911)	.04(54,122)
ΔNew FAB − ΔOld FAB	zero	.04(54,911)	.01(54,122)

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(16) The difference between the new plan ΔFAB and the old plan ΔFAB will be used to calculate the difference in the PUC normal cost:

old plan PUC NC = $(\Delta \text{Old FAB})\, \ddot{a}_{65}^{(12)}\, D_{65}/D_x$
new plan PUC NC = $(\Delta \text{New FAB})\, \ddot{a}_{65}^{(12)}\, D_{65}/D_x$
ΔPUC NC = $(\Delta \text{New FAB} - \Delta \text{Old FAB})\, \ddot{a}_{65}^{(12)}\, D_{65}/D_x$

	Smith	Brown	Green	
$\Delta(\Delta FAB)$	zero (stop working on Smith!)	.04(54,911) = 2,196.44	.01(54,122) = 541.22	
PV factor		$\ddot{a}_{65}^{(12)} D_{65}/D_{48}$ $= 8.74(1.07)^{-17}$ $= 2.7669$	$\ddot{a}_{65}^{(12)} D_{65}/D_{60}$ $= 8.74(1.07)^{-5}$ $= 6.2315$	no pre-ret decrement
ΔPUC NC		2196.44(2.7669) = 6,077.24	541.22(6.2315) = 3,372.61	$\Sigma = 9449.85$

E

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17 This is a messy Projected Unit Credit problem. It is between 1999 #15 and 1999 #16 in difficulty, and includes concepts covered in both of those problems. The key point of this problem is very carefully handling the definition of the present values for married and unmarried participants.

For married participants, the construction of the present value factor at age 65 is

$$90\%(\text{life annuity}) + (90\%)(60\%)(\text{reversionary annuity})$$
$$= .90\ddot{a}_{65}^{(12)} + .54(\ddot{a}_{65}^{(12)} - \ddot{a}_{65:65}^{(12)})$$
$$= 1.44\ddot{a}_{65}^{(12)} - .54\ddot{a}_{65:65}^{(12)}$$
$$= 1.44(8.74) - .54(6.90)$$
$$= 8.8596$$ not much subsidy for married partic!

For valuation purposes, you need to apply the % assumed married to the appropriate factor:

$$\text{PV factor at } 65 = 15\%(\text{life annuity}) + 85\%(\text{married annuity})$$
$$= .15(8.74) + .85(8.8596)$$
$$= 8.8417$$ even closer to life annuity!

Once you have this present value factor, the problem is a typical PUC problem. The expected UAL is calculated based on the standard formula:

$$eUAL_1 = (1+i)(NC_0 + UAL_0) - (\text{contrib} + \text{interest})$$

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(17) You must determine the projected final average pay to calculate both the normal cost and accrued liability at 1-1-99. Those values can be plugged into the prior formula to produce the expected UAL.

Birth date 1-1-57 1-1-99 Age 42
Hire date 1-1-87 Past service 12

Age 42 pay = 75,000
Age 64 pay = $75{,}000(1.04)^{22}$
= 177,744

1-1-99 PUC FAB = 2% (12)(177,744)
1999 PUC ΔFAB = 2% (1)(177,744)

You can simply calculate the PUC normal cost, then multiply by 12 to get the PUC accrued liability at 1-1-99.

weighted PV factor

$$\text{1-1-99 PUC NC} = .02(177{,}744)\text{"}\ddot{a}_{65}^{(12)}\text{"}\ D_{65}/D_{42}$$
$$= .02(177{,}744)(8.8417)(1.07)^{-23}$$
$$= 6{,}631$$

1-1-99 PUC AL = 12(6,631) = 79,566

$$\text{12-31-99 eUAL} = 1.07(6{,}631 + 79{,}566) - 1.035(7{,}000)$$
$$= 84{,}985 = 92{,}230 - 7{,}245$$

A

If you used compound interest to bring the contribution forward, the eUAL = 84,989.

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18 In this problem, you are given information for the Unit Credit method, which you can use to calculate the Entry Age Normal accrued liability.

The key to working this problem is knowing the third definition for the EAN accrued liability, which saves several steps:

Retrospective EAN AL: $EANC\left({}^{s}\ddot{s}_{EA:\overline{CA-EA|}}\right)$

Prospective EAN AL: $PVB - EANC\left({}^{s}\ddot{a}_{CA:\overline{RA-CA|}}\right)$

Alternative EAN AL: $PVB\left(\dfrac{{}^{s}\ddot{a}_{EA:\overline{CA-EA|}}}{{}^{s}\ddot{a}_{EA:\overline{RA-EA|}}}\right)$

In this problem, the benefits are not based on pay, so the annuity factors do not have salary scales, and the EAN normal cost is determined as a level dollar amount.

1-1-99 Age 45 Past service 15
Hire Age 30 Total service 35

$UCNC = PV(\Delta AB)$
$= PV(12)(\$25)$

$UC\ AL = PV(AB)$
$= PV(12)(\$25)(15)$
$= 10{,}125 = 15(675)$

The Unit Credit accrued liability is 15 times the Unit Credit normal cost

$EAN\ AL = PVB\left(\ddot{a}_{30:\overline{15|}} / \ddot{a}_{30:\overline{35|}}\right)$

$PVB = PV(12)(\$25)(35) = 35(675) = 23{,}625$

$EAN\ AL = 23{,}625\left(\ddot{a}_{\overline{15|}.07} / \ddot{a}_{\overline{35|}.07}\right)$ no pre-ret decrements

$= 16{,}619$

$\Delta AL = 6{,}494 = 16{,}619 - 10{,}125$ D

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19 This is a fairly confusing problem, with different pre-retirement interest rates, but the same post-retirement interest rates. With salary scales, and different assumed retirement ages, this requires very careful attention to avoid making any logical or arithmetic errors.

You need to allow for the unreduced benefit at age 62 under the old assumptions. I prefer to set up two columns of information, one for each set of assumptions:

	OLD	NEW
Pre-ret interest	7%	6%
Assumed retirement	62	65
1-1-99 Age	45	45
Past service	13	13
Future service	17	20
Total benefit service	30	30 (limited)
Age 45 pay	60,000	60,000
(ARA-1) pay	$60{,}000(1.04)^{16}$	$60{,}000(1.04)^{19}$
	$= 112{,}379$	$= 112{,}379(1.04)^3$
ARA – FAE3	$112{,}379\,(\ddot{a}_{\overline{3}\vert .04}/3)$	
	$= 108{,}112$	$121{,}611 = 108{,}112(1.04)^3$

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(19)

	OLD Assump	NEW Assump
FAE at ARA	108,112	121,611
ARA	62	65
Pre-ret interest	7%	6%
Early reduction at ARA	1.00	1.00
Projected Benefit	2%(30)(108,112)	2%(30)(121,611)
	= 64,867	= 72,967

Under the Aggregate method, the PVNC is defined as PVB - AAV. The NC is calculated as the PVNC divided by the average temporary annuity. Since the benefit is pay related, the annuity should reflect salary scale, and the NC will be calculated as a level % of pay.

	at 7%	at 6%
1-1-99 PVB	$64{,}867(\ddot{a}_{62}^{(12)})(D_{62}/D_{45})$	$72{,}967(\ddot{a}_{65}^{(12)})(D_{65}/D_{45})$
	$= 64{,}867(9.39)(1.07)^{-17}$	$= 72{,}967(8.74)(1.06)^{-20}$ Too tricky!
	$= 192{,}826$	$= 198{,}847$
PVNC	142,826	148,847
average ${}^{s}\ddot{a}_{x:\overline{RA-x}\rvert}$	${}^{s}\ddot{a}_{45:\overline{17}\rvert}$ at 7%	${}^{s}\ddot{a}_{45:\overline{20}\rvert}$ at 6%
	$= 1 + \frac{1.04}{1.07} + \ldots + \left(\frac{1.04}{1.07}\right)^{16}$	$= 1 + \frac{1.04}{1.06} + \ldots + \left(\frac{1.04}{1.06}\right)^{19}$
	$= \ddot{a}_{\overline{17}\rvert j}$ where $j = 2.88\%$	$= \ddot{a}_{\overline{20}\rvert k}$ where $k = 1.92\%$
	$= 13.6726$	$= 16.7903$
1-1-99 AGG NC	$142{,}826/13.6726$	$148{,}847/16.7903$
	$= 10{,}446$	$= 8{,}865$ $\Delta = 1{,}581$ (C)

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20 It has been rare to see select and ultimate decrements on this exam. My preferred approach is to set up the problem without the select + ultimate decrements, and then to allow for them as the final step in the solution.

The Entry Age normal cost is defined as

$$EANC = \frac{PVB_{EA}}{\ddot{a}_{EA:\overline{RA-EA}|}} \quad \text{or} \quad \frac{PVB_{EA}}{{}^{s}\ddot{a}_{EA:\overline{RA-EA}|}}$$

Since the benefit is based on pay, you need to use the second definition to calculate the normal cost as a level % of pay. If the participant's current age is different than entry age, then the normal cost would be adjusted by the salary scale for the difference in age.

Birth date	1-1-67	1-1-99 Age 32
Hire date	1-1-99	Past service 0
		Total service 33

Age 32 pay 75,000

Age 64 pay $75{,}000\,(1.05)^{32}$

$= 357{,}371$ (HUGE!)

Projected benefit $(2\%)(33)(357{,}371)$

$= 235{,}865$

Of course, there are no 401(a)(17) or 415(b) limits here.

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(20) In this problem there is no handy shortcut to determine the EANC. If there were no decrements, it would be calculated as follows

No Decrements $EANC = PVB_{EA} / {}^{s}\ddot{a}_{EA:\overline{RA-EA|}}$

No Decrements $PVB_{EA} = 235{,}865\ \ddot{a}^{(12)}_{65} (D_{65}/D_{32})$

$= 235{,}865\,(8.74)(1.07)^{-33}$

No Decrements ${}^{s}\ddot{a}_{EA:\overline{RA-EA|}} = \ddot{a}_{\overline{33|}j}$ where $1+j = \frac{1.07}{1.05} = 1.0190$

The effect of the select decrements on the PVB is straightforward if you express it in simplest terms

$D_{65}/D_{32} = {}_{33}p^{(T)}_{32}\,(1.07)^{-33}$

$= p^{(T)}_{32}\, p^{(T)}_{33}\, p^{(T)}_{34}\, p^{(T)}_{35} \cdots p^{(T)}_{64}\,(1.07)^{-33}$

$= .92\,(.96)(.98)\,(1)\cdots(1)\,(1.07)^{-33}$

$= .09282$

$PVB_{EA} = 235{,}865(8.74)(.09282)$

$= 191{,}335$

The effect on the temporary annuity is easier to see if you express it in simplest terms also:

${}^{s}\ddot{a}_{32:\overline{33|}} = 1 + \frac{1.05}{1.07} p^{(T)}_{32} + \left(\frac{1.05}{1.07}\right)^2 p^{(T)}_{32} p^{(T)}_{33} + \left(\frac{1.05}{1.07}\right)^3 p^{(T)}_{32} p^{(T)}_{33} p^{(T)}_{34} \left({}^{s}\ddot{a}_{35:\overline{30|}}\right)$

$= 1 + \frac{.92}{1.019} + \frac{.92(.96)}{(1.019)^2} + \frac{.92(.96).98}{(1.019)^3}\left(\ddot{a}_{\overline{30|}1.90\%}\right)$

$= 21.6699$

$EANC = 191{,}335/21.6699$

$= 8{,}830$

C