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SPRING 1998 EA-1B
EXAM SOLUTIONS

Revision History:

12/16/02 Corrected solutions for problem 2 - incorrect temporary annuity calculation

07/12/01 Added alternate solutions for problems 2, 7, 12, and 13

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1 The key to this problem is handling the varying salary scale (carefully). First time on this exam!

The Entry Age Normal cost can be calculated as (PVB at entry age) / ${}^{s}\ddot{a}_{EA:\overline{RA-EA}|}$ which gives the normal cost which will be level as a percentage of payroll. Today's normal cost would be calculated by multiplying that prior value by (PAY_{CA} / Pay_{EA}).

1-1-98 Age 50 Entry Age 30

$$PVB_{EA} = 12(1000)\ \ddot{a}_{65}^{(12)}\ D_{65}/D_{30}$$
$$= (12{,}000)(104.83/12)(1.07)^{-35}$$
$$= 1000\ (104.83)(.093663) = 9818.69$$

$${}^{s}\ddot{a}_{EA:\overline{RA-EA}|} = {}^{s}\ddot{a}_{30:\overline{35}|}$$
$$= 1 + (1.03/1.07)^1 + (1.03)^2/(1.07)^2 + \ldots + (1.03/1.07)^9$$
$$+ (1.03/1.07)^{10}\left[1 + (1.02/1.07) + (1.02/1.07)^2 + \ldots + (1.02/1.07)^{24}\right]$$
$$= \ddot{a}_{\overline{10}|j} + (1.03/1.07)^{10}\ \ddot{a}_{\overline{25}|k}$$
where $1+j = 1.07/1.03 = 1.0388$

$1+k = 1.07/1.02 = 1.0490$

$$= \ddot{a}_{\overline{10}|3.88\%} + .6832\ \ddot{a}_{\overline{25}|4.90\%}$$
$$= 8.4750 + .6832(14.9312)$$
$$= 18.6756$$

Handling this salary weighted annuity is one key aspect of the problem. With no pre-retirement decrements, you can resolve the value of the annuity based on a "new" interest rate.

$$EANC_{30} = 9818.69 / 18.6756 = 525.75$$
$$EANC_{50} = EANC_{30}\ (Pay_{50} / Pay_{30})$$
$$= 525.75\ (1.03)^{10}(1.02)^{10} = 861.29$$

E within "implied" range

Added 07/12/01

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2 This problem looks like a typical forecast valuation question, but it is harder to work it that way - see the next two pages. You are not given the % of pay or $\ddot{a}_{65}^{(12)}$, but you can solve for their product, based on the formula for the normal cost under Aggregate:

$$\text{Aggregate NC} = \frac{PVB - AAV}{\text{average } {}^{s}\ddot{a}_{x:\overline{RA-X}|}}$$

1-1-97 valuation

Smith is the only participant in 1997, age 52

1-1-97 $PVB = X\% \left(\ddot{a}_{65}^{(12)}\right)(D_{65}/D_{52})$ (Final pay at 64)

$D_{65}/D_{52} = (1.07)^{-13}$ with no pre-retirement decrements

Final pay $= (40{,}000)(1.05)^{12}$

$$\therefore PVB = \underbrace{X\% \left(\ddot{a}_{65}^{(12)}\right)}_{Z} (1.07)^{-13}(40{,}000)(1.05)^{12}$$

$$17{,}500 = NC = \frac{Z\,(1.07)^{-13}(40{,}000)(1.05)^{12} - 75{,}000}{{}^{s}\ddot{a}_{52:\overline{13}|}}$$

$$17{,}500\left({}^{s}\ddot{a}_{52:\overline{13}|}\right) + 75{,}000 = Z\,(1.07)^{-13}(40{,}000)(1.05)^{12}$$

$${}^{s}\ddot{a}_{52:\overline{13}|} = 1 + \frac{1.05}{1.07} + \dots + \left(\frac{1.05}{1.07}\right)^{12} = \ddot{a}_{\overline{13}|} \; 1.90\%$$

$$Z = \frac{17{,}500\left(\ddot{a}_{\overline{13}|.0190}\right) + 75{,}000}{(1.07)^{-13}(40{,}000)(1.05)^{12}}$$

$$Z = 9.3481 = \frac{17{,}500(11.6375) + 75{,}000}{.4150(40{,}000)(1.7959)}$$

Revised 12/16/02

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(2) 1-1-98 valuation

Luckily, Smith and Brown are the same age (53) ∴ which simplifies the calculations a bit.

$$\text{1-1-98 PVB} = X_{90}(\ddot{a}_{65}^{(12)})(D_{65}/D_{53})(\text{Total Pay at 64})$$
$$= Z\ (1.07)^{-12}\ (42{,}000+37{,}000)(1.05)^{11}$$
$$= 9.3481(.4440)(79{,}000)(1.7103)$$
$$= 560{,}827$$

$$\text{1-1-98 AAV} = 1.105(\text{1-1-97 AAV} + \text{1-1-97 contribution}) - 0 \text{ ben pmts}$$
$$= 99{,}450 = 1.105(75{,}000+15{,}000)$$
$$\text{PVNC} = 560{,}827 - 99{,}450$$
$$= 461{,}377$$

$$\text{average } {}^{S}\ddot{a}_{53:\overline{12|}} = \frac{42{,}000(\ddot{a}_{\overline{12|}1.90\%}) + 37{,}000(\ddot{a}_{\overline{12|}1.90\%})}{42{,}000+37{,}000} = 10.8401$$

$$\text{1-1-98 NC} = \frac{\text{PVB}-\text{AAV}}{\text{avg } {}^{S}\ddot{a}_{x:\overline{RA-x|}}}$$
$$= \frac{461{,}377}{10.8401}$$
$$= 42{,}562$$

B

Some students prefer this solution, since it relies less on theory. The amount of calculations may also be slightly less than the next two pages

Revised 07/12/01

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(THE HARD WAY)

(2) If all assumptions are met, the normal cost will remain level under most cost methods. Under the Aggregate method it is also necessary that the discounted value of the contribution for the year is equal to the prior year normal cost. This problem is the first time this additional requirement was tested!

You need to derive values for the 1-1-98 expected balance sheet to use for the 1-1-98 actual balance sheet

	1-1-97 Actual	1-1-98 Expected	1-1-98 Actual
PVB			
AAV	75,000	1.07(75,000 + 17,500) = 98,975	
PVNC			
PVE			
Earnings	40,000	40,000(1.05)	
PVE/E			
NC	17,500	17,500(1.05)	

Based on the assumed "expected" contribution of 17,500 at 1-1-97, we can write the expected NC as 17,500(1.05), which remains level as a percentage of payroll. Coincidentally, Smith's pay increased by the salary scale from 1-1-97 to 1-1-98.

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(2) The next step is to calculate the value of the annuity used to determine the normal cost. With only one participant in the expected balance sheet, we have

1-1-98 age 53

$${}^{S}\ddot{a}_{53:\overline{RA-53|}} = {}^{S}\ddot{a}_{53:\overline{12|}}$$

$$= 1 + \frac{1.05}{1.07} + \cdots + \left(\frac{1.05}{1.07}\right)^{11}$$

$$= \ddot{a}_{\overline{12|}j} \text{ where } 1+j = 1.07/1.05 = 1.0190$$

$$= 10.8401$$

Now you can complete the expected values by calculating the PVNC as 10.8401(17,500)(1.05) = 199,186, and the PVB as 199,186 + 98,975 = 298,161:

	1-1-97 Actual	1-1-98 Expected	1-1-98 Actual
PVB		298,161 (above)	$560,827 = 298,161\left(1+\frac{37,000}{42,000}\right)$
AAV		1.07(75,000+17,500) = 98,975	1.05(75,000+15,000) = 99,450
PVNC		10.8401(17,500)(1.05) = 199,186	461,377 = 560,827 − 99,450
Earn	40,000	1.05(40,000) = 42,000	
PVE/E		10.8401 (above)	10.8401 Smith age = Brown age
NC	17,500	1.05(17,500) = 18,375	42,562 = 461,377/10.8401
NCAR	43.75%	43.75%	B

Since Smith and Brown are the same age, and the benefit is a uniform % of pay, Brown's PVB equals Smith's times $\left(\frac{37,000}{42,000}\right)$. The average temporary annuity for the normal cost is unchanged because Brown is the same age as Smith. Actual contribution is used in AAV.

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3 The key points to working this problem are knowing how to handle the employee contributions, and the Aggregate cost method. In general, the employee contributions are treated similar to the actuarial value of assets. This problem is less difficult than others involving employee contributions, since you don't have any calculations of either PV of future EEC, or of refunds of EEC. You should assume that the present value of any refunds is included in the PVB.

$$\text{AGG PVNC} = \text{PVB} - \text{AAV} - \text{PVEEC}$$
$$= 210{,}000 - 25{,}000 - 23{,}000$$
$$= 162{,}000$$

There is one detail, which is calculating the average temporary annuity used for the NC:

$$\text{avg } \ddot{a}_{x:\overline{RA-x}|} = \frac{2(\ddot{a}_{55:\overline{10}|}) + 3(\ddot{a}_{53:\overline{12}|})}{5}$$
$$= \frac{2\ddot{a}_{\overline{10}|.07} + 3\ddot{a}_{\overline{12}|.07}}{5} \qquad \text{(no decrements pre-ret)}$$
$$= \frac{2(7.5152) + 3(8.4987)}{5}$$
$$= 8.1053$$

$$\text{AGG NC} = \text{PVNC} / (\text{avg } \ddot{a}_{x:\overline{RA-x}|})$$
$$= 162{,}000 / 8.1053$$
$$= 19{,}987 \qquad \text{C}$$

This problem seems a bit too easy, since there are no "tricky" aspects to it.

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4 The key to this problem is correctly calculating the final average pay benefits under the two alternate plans. One potential "trick" is that you are given 1997 compensation, NOT the 1998 compensation.

Under the Aggregate method you have these definitions

$PVNC = PVFB - AAV$

$NC = PVNC / (\text{average } {}^s\ddot{a}_{x:\overline{RA-x}|})$

1-1-98 Age 60 NRA 65 Age 59 pay 45,000

Past svc 4 Tot svc 9 Age 64 pay $54{,}749 = 45{,}000(1.04)^5$

Old plan benefit $= .75(54{,}749)\dfrac{\ddot{a}_{\overline{5}|.04}}{5}\left(\dfrac{9}{15}\right) = .75(54{,}749).9260(.60)$

$= 22{,}813$

New plan benefit $= .60(54{,}749)\dfrac{\ddot{a}_{\overline{3}|.04}}{3} = .60(54{,}749).9620$

$= 31{,}602$

$\Delta PVFB = (\Delta \text{Proj Benefit})\,\ddot{a}^{(12)}_{65}\,D_{65}/D_{60}$

$= (31{,}602 - 22{,}813)\,8.736\,(1.07)^{-5}$ no pre-ret decrement

$= 54{,}743$

$\Delta PVNC = 54{,}743$

$\Delta NC = 54{,}743 / {}^s\ddot{a}_{60:\overline{5}|}$ with only one participant

${}^s\ddot{a}_{60:\overline{5}|} = 1 + \dfrac{1.04}{1.07} + \dots + \left(\dfrac{1.04}{1.07}\right)^4$

$= \ddot{a}_{\overline{5}|j}$ where $1+j = 1.07/1.04 = 1.0288$

$= 4.7274$

$\Delta NC = \dfrac{54{,}743}{4.7274}$

$= 11{,}580$

D

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5 This problem is dependent on your ability to handle the "life insurance" death benefits. One slight twist is that the benefit is pay related, but you are told the normal cost is determined as a level $ amount.

Under Entry Age Normal, you calculate the normal cost as $PVB_{EA} / \ddot{a}_{EA:\overline{RA-EA}|}$ on the level $ approach. Be careful in handling pay for benefits:

1-1-98 Age 48 NRA 65 1997: Age 47 pay 30,000

1-1-75 Entry 25 Total svc 40 Age 64 pay $58,437 = 30{,}000(1.04)^{17}$

Projected benefit: $.02(40)(58{,}437)(\ddot{a}_{\overline{3}|.04}/3)$

$44{,}975 = .80(58{,}437).9620$

$$PVB_{EA} = 44{,}975\,\ddot{a}_{65}^{(12)}(D_{65}/D_{25}) + [10{,}000(M_{25}-M_{65}) + 40{,}000(M_{65})]/D_{25}$$

To value the insurance benefits, you can use

$M_x = D_x - d\cdot N_x$ $\quad M_{25} = D_{25} - iv(N_{25}) = 5391 - \frac{.07}{1.07}(77{,}800) = 301.28$

$A_x = 1 - d\cdot\ddot{a}_x$ $\quad M_{65} - D_{65} - iv(N_{65}) = 286 - \frac{.07}{1.07}(2{,}630) = 113.94$

$$PVB_{EA} = 44{,}975(8.74)\frac{286}{5{,}391} + \frac{10{,}000(301.28) + 30{,}000(113.94)}{5{,}391}$$

$= (112{,}420{,}314 + 3{,}012{,}804 + 3{,}418{,}318) / 5{,}391$

$= 22{,}046$

$EANC = PVB_{EA} / \ddot{a}_{EA:\overline{RA-EA}|}$

$= 22{,}046 / \ddot{a}_{25:\overline{40}|} = 22{,}046(D_{25})/[N_{25}-N_{65}]$

$= 22{,}046 / 13.9436$ E

$= 1{,}581$ within implied range 1570-1590

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6 The key to this problem is knowing how to calculate the missing annuity value $\ddot{a}_{x+1:y+1}$. In addition, you must know how to calculate the non-investment G/L:

non-Inv G/L = $eAL_1 - AL_1$

$AL_1 = 10{,}000\ \ddot{a}_{x+1} + 5000(\ddot{a}_{y+1} - \ddot{a}_{x+1:y+1})$

To derive the value of the missing annuity, you would start with the formula for a single life annuity:

$v p_x\, \ddot{a}_{x+1} = a_x$ $\qquad v p_y\, \ddot{a}_{y+1} = a_y$ $\qquad v p_{xy}\, \ddot{a}_{x+1:y+1} = a_{xy}$

$p_x = \dfrac{(1+i)(\ddot{a}_x - 1.0)}{\ddot{a}_{x+1}}$ $\qquad p_y = \dfrac{(1+i)(\ddot{a}_y - 1.0)}{\ddot{a}_{y+1}}$ $\qquad \ddot{a}_{x+1:y+1} = \dfrac{(1+i)(\ddot{a}_{xy} - 1.0)}{p_x p_y}$

$= 1.07(7.157)/7.915$ $\qquad = 1.07(9.301)/10.059$ $\qquad$ need to solve for p_x and p_y first

$= .9675$ $\qquad = .9894$

Now you can calculate the value of the annuity

$\ddot{a}_{x+1:y+1} = \dfrac{1.07(6.281)}{.9675(.9894)} = 7.0209$

Finally, you can calculate the AL_1 value

$AL_1 = 10{,}000(7.915) + 5{,}000(10.059 - 7.0209)$

$= 94{,}341$

There are two ways to calculate the eAL_1 value. In this problem, the quickest approach is using a general formula:

$eAL_1 = (1+i)(AL_0)$ − (actual benefit pmts + interest)

$AL_0 = 10{,}000\ \ddot{a}_x + 5{,}000(\ddot{a}_y - \ddot{a}_{xy})$

$= 10{,}000(8.157) + 5000(10.301 - 7.281)$

$= 96{,}670$

$\therefore eAL_1 = 1.07(96{,}670) - 1.07(10{,}000) = 92{,}737$

mortality loss $= 94{,}341 - 92{,}737 = 1{,}604$ $\qquad$ B

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(6) You can also calculate the expected accrued liability by considering all possible cases of survival of x and y. When you multiply the probabilities by the present value for each case, the total will equal the figure previously determined via formula.

In this problem, this method is much longer than the formula approach for eAL_1. It does not save any time, since you still need to derive the values of p_x, p_y and $\ddot{a}_{x+1:y+1}$:

Case: Alive	Probability	Present value	Probability * Present value
x only	$p_x q_y = .9675(.0106)$	$10{,}000\ \ddot{a}_{x+1}$	812
y only	$q_x p_y = .0325(.9894)$	$5{,}000\ \ddot{a}_{y+1}$	1,617
x and y	$p_x p_y = .9675(.9894)$	$10{,}000\ddot{a}_{x+1} + 5{,}000(\ddot{a}_{y+1} - \ddot{a}_{x+1:y+1})$	90,307
Neither	$q_x q_y = .0325(.0106)$	0	0
			92,736

Added 07/12/01

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7 This problem is similar to both 1996 #14 and 1997 #16. It is easier than some others, since you really only do calculations for one person. The reason is that Smith is a new entrant, and their retrospective accrued liability is zero, since there are no prior normal costs.

The cleanest calculation technique accumulates the accrued liability one year at a time:

$$AL_{t+1} = \left(\frac{D_t^{(T)}}{D_{t+1}^{(T)}}\right)(AL_t + NC_t)$$

$$= \left(\frac{1+i}{p_t^{(T)}}\right)(AL_t + NC_t)$$

Date	Age_x	$p_x^{(T)}$	AL_x	AL_{x+1} EANC
1-1-95	40	$.91 = 1-(.08+.01)$	zero	$\frac{1.07}{.91}(\text{zero} + 3{,}000) = 3{,}527$
1-1-96	41	$.90 = 1-[.08+.02]$		$\frac{1.07}{.90}(3{,}527 + 3{,}000) = 7{,}760$
1-1-97	42	$.92 = 1-[.06+.02]$		$\frac{1.07}{.92}(7{,}760 + 3{,}000) = 12{,}515$

D

A more difficult method of solution is shown on the next page - but most students would choose this method, since it has fewer calculations.

Revised 07/12/01

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(THE HARD WAY)

(7) There are two keys to working this problem:
(i) ignore Smith, whose accrued liability is zero,
(ii) don't try to calculate N_x values.

The accrued liability under an individual cost method can be calculated using either a prospective or retrospective formula. You can't easily use the prospective approach, and the retrospective formula will be preferable:

$$\text{Green } AL = EANC(\ddot{s}_{EA:\overline{CA-EA}|}) \qquad \text{(on level \$ EANC)}$$

$$= 3000\ \ddot{s}_{40:\overline{3}|}$$

$$= 3000(N_{40} - N_{43})/D_{43}$$

$$= 3000\,(D_{40} + D_{41} + D_{42})/D_{43}$$

$$= 3000\left(\frac{v^{40}l_{40}^{(T)} + v^{41}l_{41}^{(T)} + v^{42}l_{42}^{(T)}}{v^{43}l_{43}^{(T)}}\right)$$

$$= 3000\left(\frac{(1+i)^3}{{}_3p_{40}^{(T)}} + \frac{(1+i)^2}{{}_2p_{41}^{(T)}} + \frac{1+i}{p_{42}^{(T)}}\right)$$

Since you are given values for $q_x^{(w)}$ and $q_x^{(d)}$, you can calculate $p_x^{(T)} = 1 - q_x^{(T)} = 1 - q_x^{(w)} - q_x^{(d)}$. This depends on there being no other pre-retirement decrements

$$\text{Green } AL = 3000\left(\frac{(1.07)^3}{p_{40}^{(T)}p_{41}^{(T)}p_{42}^{(T)}} + \frac{(1.07)^2}{p_{41}^{(T)}p_{42}^{(T)}} + \frac{1.07}{p_{42}^{(T)}}\right)$$

$$= 3000\left(\frac{(1.07)^3}{.91(.90).92} + \frac{(1.07)^2}{.90(.92)} + \frac{1.07}{.92}\right)$$

$$= 3000(1.6258 + 1.3827 + 1.1630)$$

$$= 12{,}515$$

D

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8 There are two keys to working this problem:
(i) understanding how to calculate the mortality G/L
(ii) being careful to separate gains at some ages from losses at other ages.

This problem has not been on the exam for about 15 years. An earlier similar problem was #37 in 1982.

The general formula for a non-investment G/L is

non-inv G/L = $eAL_1 - AL_1$

The key to the problem is calculation of eAL_1:

$$eAL_1 = (1+i)(AL_0 + NC_0) - (\text{actual benefit payments} + \text{interest})$$
$$= (1+i)(p_x^{(T)}/p_x^{(T)})(AL_0 + NC_0) - \emptyset \quad \text{for actives}$$
$$= \frac{D_x^{(T)}}{D_{x+1}^{(T)}}(AL_0 + NC_0)\, p_x^{(T)}$$

Since the retrospective accrued liability equals the accumulation of prior normal costs with interest and survivorship, we can rewrite this as

$$eAL_1 = (AL_1)\, p_x^{(T)}$$

To work the problem, we must assume that the only pre-retirement decrement is mortality, since that allows calculation of $p_x^{(T)}$:

Age	$q_x^{(d)}$	1-1-97 Lives	eAL_1	1997 deaths	AL_1	$\Delta = G/L$	
30	.004	2,000	.996(2,000)(1,500)	4	1,996(1,500)	-4(1,500)	Loss
40	.005	3,000	.995(3,000)(4,800)	10	2,990(4,800)	-5(4,800)	Loss
50	.008	1,000	.992(1,000)(18,000)	10	990(18,000)	2(18,000)	Gain
						6,000	Gain

(D)

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9 The keys to working this problem are twofold:
(i) knowing how to adjust the UAL for assump change,
(ii) reflecting the change in PVE/E for the normal cost.

Under the FIL cost method, you adjust the UAL for changes in benefits or assumptions by the change in the Entry Age Normal accrued liability. You can determine the EAN accrued liability by the prospective formula:

$AL = PVFB - PVEANC = PVFB - EANC({}^{s}\ddot{a}_{x:\overline{RA-x}|})$

This can be tricky with multiple participants at different ages, but we only have one participant!

<u>Original Assumptions</u>

FIL PVNC = PVB − UAL − AAV = 12,200 − 1,300 − 6,300
= 4,600

FIL NC = PVNC / (avg ${}^{s}\ddot{a}_{x:\overline{RA-x}|}$)
460 = 4,600 / (PVE/E) ⇒ PVE/E = 10.00 ⇒ PVE = 70,000
∴ Earn = 7,000

EA NC = 400

EAN AL = PVFB − PVEANC
= 12,200 − 400(10.00) = 8,200

<u>Revised Assumptions</u>

PVE = 77,000 ⇒ PVE/E = (77,000/7,000) = 11.00

EAN AL = PVFB − PV EANC
= 14,200 − 400(11.00) = 9,800

Δ EAN AL = 1,600 = 9,800 − 8,200 due to assumption change

new FIL UAL = 1,600 + 1,300 = 2,900 " " " "

FIL PVNC = PVB − UAL − AAV
= 14,200 − 2,900 − 6,300 = 5,000

FIL NC = PVNC / (PVE/E)
= 5,000 / 11.00 = 454.55 (B)

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10 This is almost identical to the only other Level Income question, which is #6 on 1984 exam. The key point is interpretation of the statement "Actuarial equivalence is based on the plan's early retirement factors."

In general, if the level income benefit is actuarially equivalent to the early retirement benefit,

$$ERB\ \ddot{a}_x^{(12)} = LIB\ \ddot{a}_x^{(12)} - PIA\left(\frac{D_{x+z}}{D_x}\right)\ddot{a}_{x+z}^{(12)}$$

LIB is the initial amount of the level income benefit, which reduces by the PIA amount at age x+z. You can rewrite this formula as

$$LIB = ERB + PIA\ \frac{D_{x+z}\ \ddot{a}_{x+z}^{(12)}}{D_x\ \ddot{a}_x^{(12)}}$$

$$= ERB + PIA\left(\frac{N_{x+z}^{(12)}}{N_x^{(12)}}\right)$$

The term $N_{x+z}^{(12)}/N_x^{(12)}$ is an actuarial equivalent reduction factor based on retirement ages x and x+z. For this problem, you can calculate the participant's early retirement benefit

$$ERB = 600\,(1 - 6\%(5) - 3\%(3))$$ using plan formula

$$= 366 = 600\,(.61)$$

$$LIB = 366 + 400\left(N_{62}^{(12)}/N_{57}^{(12)}\right)$$

The key to the problem is how you calculate the factor in parentheses. It is not correct to use 5 years of reductions based on the plan formula, since it is based on NRA 65. Rewrite the "N/N" terms in terms of NRA 65 first:

$$LIB = 366 + 400\left(\frac{N_{65}^{(12)}/N_{57}^{(12)}}{N_{65}^{(12)}/N_{62}^{(12)}}\right)$$

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(10) Now you have expressed the reductions as a ratio of actuarial reductions from age 65. You can replace the numerator and denominator of the fraction with the reductions from 65 based on the plan early retirement factors

$$LIB = 366 + 400\left(\frac{1-6\%(5)-3\%(3)}{1-6\%(3)}\right)$$

$$= 366 + 400\left(\frac{.61}{.82}\right)$$

$$= 663.56$$

C

If you try other ways to calculate the factor, you might get lucky. Neither of these approaches is correct:

$366 + 400\,(1-6\%(5)) = 366 + 400(.70) = 646$
based on 5 years of reduction

$366+400\,(1-6\%(2)-3\%(3)) = 366+400(.79) = 682$
based on 5 years from 62 down to 57

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11 For side fund problems, you use the cash surrender value at ARA to fund part of the pension benefit. Then apply the cost method to the remainder of the benefit, which produces the "side fund" normal cost. In some problems you may be asked to calculate the "total normal cost" which is the sum of the insurance premium and the side fund normal cost.

Under the ILP method, the normal cost at plan inception funds the total PVB over future service. The PVB is calculated for the net benefit not provided from the insurance.

Projected benefit	$.4(54{,}000) = 21{,}600$
Total insurance	$75(21{,}600/12) = 135{,}000$
CSV at ARA 65	$135(350) = 47{,}250$
Benefit funded by CSV	$47{,}250/8.74 = 5{,}406$
Net benefit for side fund	$21{,}600 - 5{,}406 = 16{,}194$

ILP normal cost: $\dfrac{PVB_x}{\ddot{a}_{x:\overline{RA-x}|}}$

$$= \frac{16194\ \ddot{a}^{(12)}_{65}\ D_{65}/D_{41}}{\ddot{a}_{41:\overline{24}|}}$$

$$= \frac{16194\ \ddot{a}^{(12)}_{65}}{\ddot{s}_{41:\overline{24}|}}$$

$$= \frac{16{,}194\,(8.74)}{\ddot{s}_{\overline{24}|.07}}$$ no pre-ret decrements

$= 141{,}534/62.2490$

$= 2{,}274$ (D)

This is one of the less "esoteric" side fund problems

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12 To solve this problem, you need to know how calculations are done under Individual Level Premium, Unit Credit, and Attained Age Normal.

You can use the given ILP normal cost value to solve for $\ddot{a}_{65}^{(12)}$:

$$\text{ILP NC} = \frac{PVB_x}{\ddot{a}_{x:\overline{RA-x}|}} = (10\text{ ees})\left[\frac{\text{Proj benefit } \ddot{a}_{65}^{(12)}(D_{65}/D_{45})}{\ddot{a}_{45:\overline{20}|}}\right]$$

$$= 21{,}508 = 10\left[\$30(12)30\,\ddot{a}_{65}^{(12)} / \ddot{s}_{\overline{20}|.07}\right]$$ no pre-ret decrements

$$\ddot{a}_{65}^{(12)} = \frac{21{,}508\,(43.8652)}{10(30)(12)(30)} = 8.7357$$

Under AAN, the initial UAL equals the UAL under Unit Credit.

$$\text{UC AL} = PV\,(\text{Accrued benefit})$$
$$= (10\text{ ees})(\$30)(12)(10)\,\ddot{a}_{65}^{(12)}\,D_{65}/D_{45}$$
$$= 300(120)\,8.7357\,(1.07)^{-20}$$ no pre-ret decrements
$$= 81{,}269$$

AAN is a typical aggregate-type cost method. The normal cost is calculated using the average temporary annuity to ARA. This is easy, since all ten ees are the same age:

$$PVB = (30/10)(\text{UC AL}) = 243{,}806$$
$$\text{AAN PVNC} = PVB - UAL - AAV$$
$$= 243{,}806 - (81{,}269 - 0) - 0 = 162{,}537$$
$$\text{avg } \ddot{a}_{x:\overline{RA-x}|} = \ddot{a}_{\overline{20}|.07} = 11.3356$$
$$\text{AAN NC} = PVNC / (\text{avg annuity})$$
$$= 14{,}339 = 162{,}537/11.3356$$

B

$$\text{Contribution} = 14{,}339 + \frac{81{,}269}{\ddot{a}_{\overline{30}|.07}} = 14{,}339 + \frac{81{,}269}{13.2777} = 20{,}459$$

Added 07/12/01

Spring 1998 EA-1B

(12) A very smart student suggested an alternative, much shorter solution. This is something that makes sense in retrospect, but would be hard to come up with under exam conditions.

You are told that the ILP normal cost is 21,508. This funds the cost of the benefit over 20 future service years, starting at age 45 at 1-1-98.

The Attained Age Normal cost method would have developed a normal cost differently. Each participant has 10 past service years and 20 future service years at 1-1-98. Conceptually, the normal costs will represent (20/30) of the funding, and the past service costs will represent (10/30) of the funding:

$$AANC = (20/30)\ ILP\ NC = (2/3)(21{,}508)$$
$$= 14{,}339$$

Amazingly, that is the correct value!

The past service costs are amortized over 30 years (as stated in the problem). We can adjust the ILP NC to produce the funding of the past service, which is everything else:

$$UAL\ funding = (10/30)\ ILP NC \left(\frac{\ddot{a}_{\overline{20}|.07}}{\ddot{a}_{\overline{30}|.07}} \right)$$
$$= (1/3)\ (21{,}508)(11.3356/13.2777)$$
$$= 6{,}121$$

Total plan cost $= 6{,}121 + 14{,}339 = 20{,}459$ (B)

Why does this work? Because it is the first year of the plan, and all the employees are clones.

Added 07/12/01

Spring 1998 EA-1B

13 There are two ways to work this problem - a long way, and a short way. There is a key point to this problem. When you're calculating gains or losses, you typically determine the expected accrued liability and compare to the actual:

non-inv G/L = $eAL_1 - AL_1$

$eAL_1 = (1+i)AL_0 - (\text{actual BP} + \text{interest})$

The complicating factor in this problem is the certain and life annuity for Smith. If you think about it, the certain portion of the annuity can't contribute any gain or loss. The shortcut is to totally ignore the certain annuity in the calculations. The long way (shown on the next page) is to do calculations of the benefit payments plus interest, and the certain and life annuities.

$\ddot{a}_{65}^{(12)} = \ddot{a}_{65} - \frac{11}{24} = 8.736$

To ignore the certain period for Smith means the 1-1-97 accrued liability is a deferred life annuity, and the 1-1-98 accrued liability is zero:

	Smith	Brown
1-1-97 age	60	50
PVB	$12(500)(D_{65}/D_{60})\ddot{a}_{65}^{(12)}$	$12(600)(D_{65}/D_{50})\ddot{a}_{65}^{(12)}$
	$= 6{,}000(5.7115)$	$= 7{,}200(2.6550)$
	$= 34{,}269$	$= 19{,}116$
actual BP	zero (ignore!)	zero
1-1-98 age	61	51
PVB	zero (ignore!)	$12(600)(D_{65}/D_{51})\ddot{a}_{65}^{(12)}$
		$20{,}570 = 7{,}200(2.8569)$

$eAL_1 = 1.07(34{,}269 + 19{,}116) = 57{,}122$

$G/L = 20{,}570 - 57{,}122 = 36{,}552$ Gain

B

Revised 07/12/01

Spring 1998 EA-1B

(THE HARD WAY)

(13) The key to working this problem is that, even though Smith dies at 12/31/97, there is still a liability at 1/1/98 due to the certain period. You also need to know how to calculate the non-investment gain or loss:

non-inv G/L = $eAL_1 - AL_1$

$eAL_1 = (1+i)(AL_0)$ − (actual BP + interest)

There is much extraneous information in the problem that you can ignore about early retirement benefits and normal + optional forms. Since the employees are retired and vested already, you don't use this information.

AL_0 Calculation − Smith | Brown

1-1-97 age: 60 | 50

PVB $12(500)(\ddot{a}^{(12)}_{\overline{5}|} + {}_{5|}\ddot{a}^{(12)}_{60})$ | $12(600)\ddot{a}^{(12)}_{65}D_{65}/D_{50}$

$\ddot{a}^{(12)}_{\overline{5}|.07} = \frac{1}{12}(\ddot{a}_{\overline{60}|j})$ where $(1+j)^{12} = 1.07 \Rightarrow j = .565\%$

$= \frac{1}{12}(51.0487) = 4.2541$ | $\ddot{a}^{(12)}_{65} = \ddot{a}_{65} - \frac{11}{24} = 8.736$

PVB $6000(4.2541 + \ddot{a}^{(12)}_{65}(D_{65}/D_{60}))$ | $7200\,\ddot{a}^{(12)}_{65}(D_{65}/D_{50})$

$= 6000(4.2541 + 5.7115)$ | $= 7200(2.6550)$

$= 59{,}794$ | $= 19{,}116$

actual BP+i: $500[12 + \frac{.07}{12}(12+11+\dots+1)]$ | 0

$= 500(12)(1 + \frac{13}{24}(.07))$

$= 6228$

AL_1 Calculation

PVB $12(500)\ddot{a}^{(12)}_{\overline{4}|.07}$ | $12(600)\ddot{a}^{(12)}_{65}D_{65}/D_{51}$

$= 6000(42.1718/12)$ | $= 19{,}116(D_{50}/D_{51})$

$= 21{,}086$ | $= 20{,}570$

$eAL_1 = 1.07(59{,}794 + 19{,}116) - 6{,}228 = 78{,}206$

non-inv G/L $= 78{,}206 - (21{,}086 + 20{,}570) = 36{,}550$ gain (B)

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14 If all assumptions are met, the normal cost will remain level under most cost methods. Under the Aggregate method, it is also necessary that the discounted value of the contribution for the year is equal to the prior year normal cost. See #2 on this 1998 exam to see what happens when this condition is NOT satisfied.

You need to derive some values at 1-1-97 to set up the 1-1-98 expected and actual balance sheets. I'll show the quickest method of solution first:

	1-1-97 Actual	1-1-98 Expected	1-1-98 Actual
PVB	4,087,881		$1.07\left(\frac{1.04}{1.05}\right)(4{,}087{,}881)$ = 4,332,375
AAV	500,000		1.09 (500,000 + 178,328) = 739,378
PVNC	4,087,881 − 500,000 = 3,587,881		4,332,375 − 739,378 = 3,592,998
PVE			
Earn	1,000,000	1.05(1,000,000) = 1,050,000	
PVE/E	3,587,881/178,328 = 20.1196	$\frac{1.07}{1.05}(20.1196-1)$ = 19.4837	19.4837 salary scale ⇒ PVE, Earn same value
NC	178,328	1.05(178,328) = 187,244	184,410
NCAR	17.83%	17.83%	

C

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(14) If you want to be 100% sure that you are working the problem correctly, you can fill in the expected balance sheet completely.

Instead of <u>assuming</u> that the normal cost will increase by 5%, you can <u>prove</u> that it will:

	1-1-97 Actual	1-1-98 Expected	
PVB	4,087,881	1.07(4,087,881) = 4,374,033	
AAV	500,000	1.07(500,000+178,328) = 725,811	
PVNC	4,087,881-500,000 = 3,587,881	4,374,033-725,811 = 3,648,222	also equals 1.07(3,587,881-178,328)
NC	178,328		3,648,222/19.4837 = 187,244
PVE/E	3,587,881/178,328 = 20.1196		20,457,930/1,050,000 = 19.4837
Earn	1,000,000	1.05(1,000,000) = 1,050,000	
PVE	20.1196(1,000,000) = 20,119,561	1.07(20,119,561-1,000,000) = 20,457,930	
NCAR	17.83%		187,244/1,050,000 17.83%

The key point is that the contribution under the Aggregate method must equal the normal cost (in discounted value at 1/1). For other methods which have a UAL, the contribution would not affect the subsequent year's normal cost - up to a point!

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15 This problem has been on the exam many times, such as #11 in 1997. The key to the problem is interpreting the impact of the plan change on the accrued liability.

Prior to the amendment, present values will be based only on the life annuity. After the plan is amended, the accrued liability will be based on a different present value factor for the 80% of the participants assumed to retire married.

UC AL = PV of accrued benefit

1/1/98 Age 45 Accrued benefit = \$35(12)(20)

Past svc 20 = 8,400

	Before Amendment	After Amendment
Age 65 PV factor	$\ddot{a}_{65}^{(12)}$	$.2(\ddot{a}_{65}^{(12)})$ $+.8(\ddot{a}_{65}^{(12)} + .5(\ddot{a}_{65}^{(12)} - \ddot{a}_{65:65}^{(12)}))$ $= \ddot{a}_{65}^{(12)} + .4(\ddot{a}_{65}^{(12)} - \ddot{a}_{65:65}^{(12)})$
PVAB	$8400\ddot{a}_{65}^{(12)} D_{65}/D_{45}$	$8400(PVF)\,D_{65}/D_{45}$

$$\begin{aligned}\Delta PVAB &= 8400(.4)(\ddot{a}_{65}^{(12)} - \ddot{a}_{65:65}^{(12)})D_{65}/D_{45} \\ &= 8400(.4)(8.736 - 6.549)(1.07)^{-20} \\ &= 8400(.4)(2.187)(.2584) \\ &= 1898.95\end{aligned}$$

no pre-ret decrements

B

This is relatively less complicated than some other earlier incarnations of this problem!

Spring 1998 EA-1B

16 The key to this problem is knowing how to calculate the normal cost under Projected Unit Credit. This problem is the first one since # 26 in 1983 that uses a "career average" benefit.

1-1-98 Age 30   Age 30 Pay = 50,000   Total service 35

Under PUC, the normal cost is defined based on the "funding" accrued benefit (FAB) instead of the actual AB:

$\text{PUC NC} = (\Delta FAB)_x \ddot{a}_{65}^{(12)} D_{65}/D_x$ $\text{PUC AL} = FAB_x \ddot{a}_{65}^{(12)} D_{65}/D_x$

$FAB = (\text{Projected Benefit}) \dfrac{\text{"PS"}}{\text{"TS"}}$ years of service are weighted by rates of benefit accrual

The first step is calculation of the projected benefit

Proposal A

$$\text{Proj Ben} = \underset{\text{age } 30}{.02(50{,}000)} + \underset{\text{age } 31}{.02(50{,}000)(1.03)} + \ldots + \underset{\text{age } 64}{.02(50{,}000)(1.03)^{34}}$$

$$= .02(50{,}000)[1 + 1.03 + \ldots + (1.03)^{34}]$$

$$= .02(50{,}000)\, s_{\overline{35}|.03} = 1000\left(\frac{62.2759}{1.03}\right) = 60{,}462$$

Proposal B

$$\text{Proj Ben} = .015(35)(\text{Pay at } 64)(\ddot{a}_{\overline{5}|.03}/5)$$

$$= .015(35)(50{,}000)(1.03)^{34}(\ddot{a}_{\overline{5}|.03}/5)$$

$$= .015(35)(50{,}000)(2.7319)(4.7171/5) = 67{,}655$$

Proposal A

$$\text{1-1-98 } FAB = 60{,}462\left(\frac{0(.02)}{35(.02)}\right) = 0$$

$$\text{1-1-99 } FAB = 60{,}462\left(\frac{1(.02)}{35(.02)}\right) = 1727.49$$

$$NC = 1727.49\, \ddot{a}_{65}^{(12)} D_{65}/D_{30}$$

$$= 1727.49(8.7358)(94{,}414/1{,}261{,}611) = 1129.35$$

$$\ddot{a}_{65}^{(12)} = \ddot{a}_{65} - \frac{11}{24} = \frac{868{,}052}{94{,}414} - \frac{11}{24} = 8.7358$$

Proposal B

$$\text{1-1-98 } FAB = 67{,}655\left(\frac{0(.015)}{35(.015)}\right) = 0$$

$$\text{1-1-99 } FAB = 67{,}655\left(\frac{1(.015)}{35(.015)}\right) = 1933.00$$

$$NC = 1933.00(8.7358)(94{,}414/1{,}261{,}611) = 1263.70$$

$$\Delta NC = 134.35 = 1263.70 - 1129.35$$

A

within implied range

Spring 1998 EA-1B

17 This is a very difficult problem which tests your ability to do Projected Unit Credit calculations with retirement decrements. Luckily there is only one retirement decrement prior to NRA, otherwise this would be much longer!

Unlike the prior problem, the rates of benefit accrual change based on years of service. Since this problem does not involve a "career average" benefit, we can use a shortcut to calculate the FAB: simply apply the benefit formula to past service, and use projected final average earnings. The complication here is the retirement at both age 62 and age 65.

	Smith	Brown
1-1-98 age	53	60
current pay	50,000	60,000
past service	21	31
Projected pay at 64	$50{,}000(1.04)^{11}$ $=76{,}973$	$60{,}000(1.04)^{4}$ $=70{,}192$
Projected pay at 61	$76{,}973(1.04)^{-3}$ $=68{,}428$	$70{,}192(1.04)^{-3}$ $=62{,}400$
FAB "at age 65"	$76{,}973(.02(20)+.01)$ $=31{,}559$	$70{,}192(.02(20)+.01(10)+0)$ $=35{,}096$
FAB "at age 62"	$68{,}428(.02(20)+.01)(1-.09)$ $=25{,}531$	$62{,}400(.02(20)+.01(10))(1-.09)$ $=28{,}392$

Key point here - don't forget the early retirement factor at age 62

PUC AL: $.3(FAB_{62})\ddot{a}_{62}^{(12)}D_{62}/D_x + .7(FAB_{65})\ddot{a}_{65}^{(12)}D_{65}/D_x$

Smith	Brown
$= .3(25{,}531)(9.39)(1.07)^{-9}$	$= .3(28{,}392)(9.39)(1.07)^{-2}$
$+ .7(31{,}559)(8.74)(1.07)^{-12}$	$+ .7(35{,}096)(8.74)(1.07)^{-5}$
$= 39{,}120 + 85{,}728$	$= 69{,}858 + 153{,}089$
$= 124{,}848$	$= 222{,}947$

A

$\Sigma = 347{,}795$

Spring 1998 EA-1B

18 The key to this problem is calculating the early retirement benefit under the "special program". You must calculate the Entry Age Normal accrued liability as an active employee at 1/1/98, and compare that to the present value of benefits based on retirement at 1/1/98.

There is a shortcut formula for calculating the EAN accrued liability, which is equivalent to the usual prospective / retrospective formulas:

$$EANAL = PVB_{CA}\left(\frac{{}^{s}\ddot{a}_{EA:\overline{CA-EA}|}}{{}^{s}\ddot{a}_{EA:\overline{RA-EA}|}}\right)$$

The derivation is shown in # 11 on the 1990 exam.

1-1-98 Age 57 NRA 65 Entry age 40

Past svc 17 Total svc 25

Accrued ben 12(10)(17) = 2,040 Proj ben 12(10)(25) = 3,000

$$\text{Active } PVB_{CA} = 3000\,\ddot{a}_{65}^{(12)} D_{65}/D_{57} = 3000(8.6106)(29/56) = 13,377.23$$

$$\ddot{a}_{65}^{(12)} = \ddot{a}_{65} - 11/24 = 8.6106 = (263/29) - 11/24$$

$$\text{Active EAN AL} = 13,377.23\ \frac{(N_{40}-N_{57})/D_{40}}{(N_{40}-N_{65})/D_{40}} = 13,377.23\left(\frac{2,561-604}{2,561-263}\right) = 11,392.19$$

special early retirement

$$\text{Retired ERB} = 12(10)(17+3)\left(1-\tfrac{5}{15}\right) = 2,400(.6667) = 1,600$$

regular early retirement

versus $12(10)(17)\left(1-\frac{5}{15}-\frac{3}{30}\right)$

$$\text{Retired PVB} = 1,600\,\ddot{a}_{57}^{(12)} = 1600\left[(N_{57}/D_{57})-(11/24)\right] = 1600(10.3274) = 16,524$$

$\Delta AL = 5,132 = 16,524 - 11,392$

D

Spring 1998 EA-1B

19. The key to this problem is understanding how to derive the value of the average temporary annuity to calculate the normal cost at 1-1-98. One potentially confusing aspect of the problem is that you can't separately determine values for UAL and PVB, and you can't "check" the values for the 1-1-98 expected balance sheet.

You need to derive values in the 1-1-98 expected balance sheet to complete the 1-1-98 actual balance sheet:

	1-1-97 Actual	1-1-98 Expected	1-1-98 Actual
PVB			same as expected + 4% salary increase
UAL			same as expected
AAV	1,500,000	includes same contribution value; $1.07(1{,}500{,}000)+200{,}000\left(1+\frac{9}{12}(.07)\right)$ = 1,815,500	$1.10(1{,}500{,}000)+200{,}000\left(1+\frac{9}{12}(.10)\right)$ = 1,865,000
PVNC			$\Delta PVNC = -49{,}500$
PVE	18,000,000		
Earn	1,000,000	1.04(1,000,000) = 1,040,000	
PVE/E	18.0	$\frac{1.07}{1.04}(18.0-1.0)$ = 17.4904	17.4904 same value
NC	80,000	1.04(80,000) = 83,200	$83{,}200-\frac{49{,}500}{17.4904}=80{,}370$ (D)

The PVE at 1-1-98 in the expected balance sheet can be calculated as 1.07(18,000,000 − 1,000,000) = 18,190,000. The resulting PVE/E ratio is 18,190,000/1,040,000 = 17.4904. This corresponds to the formula shown above.

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20 The key to working this problem is realizing that Brown's initial layer of ILP normal cost was set up at hire date of 1-1-96, while Smith's initial layer was set up at the plan inception date of 1-1-95. Both participants have a new layer of normal cost set up at 1-1-98 due to the plan change. In general, each new layer of benefits is funded prospectively over future service, and creates a new layer of normal cost under the ILP method.

	Smith	Brown
Hire date	1-1-93	1-1-96
Plan inception	1-1-95	1-1-95
Initial NC layer	1-1-95	1-1-96
Age at date ↗	60	51
Hire age	58	51
Total service	7	14
\$10 proj benefit	$10(12)(7)=840$	$10(12)(14)=1{,}680$
Initial ILP NC	$\dfrac{840\ddot{a}_{65}^{(12)}D_{65}/D_{60}}{\ddot{a}_{60:\overline{5}|}}$	$\dfrac{1680\ddot{a}_{65}^{(12)}D_{65}/D_{51}}{\ddot{a}_{51:\overline{14}|}}$ no pre-ret decrements
	$=840\ddot{a}_{65}^{(12)}/\ddot{s}_{\overline{5}|.07}$	$=1680\ddot{a}_{65}^{(12)}/\ddot{s}_{\overline{14}|.07}$
	$=840(8.736)/6.1533$	$=1680(8.736)/24.1290$
	$=1192.57$	$=608.25$
1-1-98 plan change		
Δ proj Benefit	$2(12)(7)=168$	$2(12)(14)=336$
1-1-98 age	63	53
New layer ILP NC	$\dfrac{168\ddot{a}_{65}^{(12)}D_{65}/D_{63}}{\ddot{a}_{63:\overline{2}|}}$	$\dfrac{336\ddot{a}_{65}^{(12)}D_{65}/D_{53}}{\ddot{a}_{53:\overline{12}|}}$
	$=168\ddot{a}_{65}^{(12)}/\ddot{s}_{\overline{2}|.07}$	$=336\ddot{a}_{65}^{(12)}/\ddot{s}_{\overline{12}|.07}$
	$=662.62$	$=153.35$
Total ILP NC	1855.20	761.60

E

$\Sigma = 2616.80$