SoftwarePolish

Rick Groszkiewicz
2964 Nestle Creek Drive
Marietta, GA 30062-4857

Voice/fax (770) 971-8913

email: rickg@softwarepolish.com

SPRING 1996 EA-1B EXAM SOLUTIONS

Revision History:
02/12/00 Corrected problem 15 - wrong annuity value
03/08/99 Corrected problem 8 - had certain only annuity instead of certain and life annuity

Spring 1996 EA-1B

1 For a typical Projected Unit Credit problem, you can calculate the accrued liability as the present value of the "funding accrued benefit": $FAB(N_{65}^{(12)}/D_x)$. The definition of the "funding accrued benefit" is that the projected benefit should be multiplied by the ratio of past service to total service, where years of service are weighted by the rates of benefit accrual.

This problem requires you to allow for retirement decrements in the calculation of the accrued liability. You must calculate a separate FAB at each retirement age. For a final average pay related plan, you can simplify the FAB by calculating projected pay to the exit age (and final average pay) and apply the benefit formula based on past service.

1-1-96 Age 50 — Age 50 pay = 50,000

Past svc 12 — Age 64 pay = 86,584 = $50{,}000(1.04)^{14}$

Age 64 FAE3 = 83,296 = $86{,}584(\ddot{a}_{\overline{3}|}^{4\%}/3)$

Age 61 FAE3 = 74,050 = $83{,}296(1.04)^{-3}$

(1) t	(2) Age $50+t$	(3) v^t	(4) (T) ${}_tp_{50}$	(5) (r) q_{50+t}	(6) Early Retirement Reduction	(7) FAB_{50+t}	(8) $\ddot{a}_{50+t}^{(12)}$	(3)(4)(5)(6)(7)(8) Product
12	62	$(1.07)^{-12}$	1.0	.2	.91	74,050(.02)(12)	9.39	13,486
13	63	–	.8	–	.94	–	–	–
14	64	–	.8	–	.97	–	–	–
15	65	$(1.07)^{-15}$	.8	1.0	1.00	83,296(.02)(12)	8.74	50,662
								64,148

C

Spring 1996 EA-1B

2 This is a projected valuation problem. The solution is based on the fact that, if all assumptions are met during a year, the normal cost must be level as a percentage of pay from 1/1/95 to 1/1/96.

You should set up three columns of information to solve this problem. One has the actual 1/1/95 valuation results, one has the expected 1/1/96 results, and the third would have the actual 1/1/96 results.

One aspect of the Aggregate cost method is that the normal cost will only be level from year to year if the beginning of the year contribution is equal to the normal cost. In this problem, you should assume the contribution for 1995 was equal to the normal cost, and use that to set up the 1/1/96 balance sheet. At the end of the problem, you have to solve for the asset value at 1/1/96.

From the 1/1/95 valuation results, you must solve for the earnings and present value of earnings:

PUNC = 1,200,000 = 1,500,000 - 300,000

PVE/E = 12.50 = 1,200,000 / 96,000

Assume 1/1/95 Pay = 1,000,000 (arbitrary)

then 1/1/95 PVE = 12,500,000

NCAR = 9.60% = 96,000 / 1,000,000

(2)

	1-1-95 Actual	1-1-96 Expected	1-1-96 "Actual" (Plan change, Salary losses)
PV benefits	1,500,000	1.07(1,500,000) = 1,605,000	$= 1{,}605{,}000\left(\frac{70}{60}\right)\left(\frac{1.06}{1.04}\right)$ = 1,908,510
AAV	300,000	1.07(300,000 + 96,000 assumed) = 423,720	423,720
PVNC	1,200,000	1,181,280	1,484,790
Earnings	1,000,000	1.04(1,000,000) = 1,040,000	(don't bother to adjust both of these
PV Earnings	12,500,000	1.07(12,500,000 - 1,000,000) = 12,305,000	for salary scale loss, get same PVE/E!)
PVE/E	12.5	11.8317	11.8317
Normal cost	96,000	99,840	125,493
NCAR	9.60%	9.60%	

At this point, based on the expected contribution of 96,000, the normal cost is 125,493. You are told that the final 1/1/96 normal cost is 122,650. The resulting PVNC should be (122,650)(11.8317) = 1,451,162. Compared to the PVNC of 1,484,790 in the third column above, there must have been an asset gain equal to 1,484,790 - 1,451,162 = 33,628. The final asset equals 423,720 + 33,628 = 457,348

D

Spring 1996 EA-1B

3 For retirement age gain and loss problem, the gain or loss is simply calculated as the difference between the accrued liability as an active employee and the accrued liability as a retiree, which is the present value of benefits. You do not consider any survival gain or loss that occurred during 1996 in this problem.

The active entry age normal accrued liability can be calculated the quickest using this expression:

$$\text{EAN AL} = PVB_{CA}\left(\frac{{}^{s}\ddot{a}_{EA:\overline{CA-EA}|}}{{}^{s}\ddot{a}_{EA:\overline{RA-EA}|}}\right) \quad \text{for pay related benefits}$$

Since this plan has benefits that are not pay related, the calculation is done using annuities with no salary scale.

1-1-96 age 55 Hire age 29 Past service 26 Total service 36

Projected benefit = \$25(12)(36) = 10,800

$$\text{Active AL} = \left(10{,}800\,\ddot{a}_{65}^{(12)}\,D_{65}/D_{55}\right)\left(\ddot{a}_{29:\overline{26}|}/\ddot{a}_{29:\overline{36}|}\right)$$

$$= 10800(8.74)(1.07)^{-10}\left(\ddot{a}_{\overline{26}|.07}/\ddot{a}_{\overline{36}|.07}\right) \quad \text{pre-retirement no decrements}$$

$$= 43{,}532 = 10800(8.74)(.5083)(12.6536/13.9477)$$

The retiree calculation is mostly a benefit calculation problem:

Accrued benefit = \$25(12)(26) = 7,800

Early retirement benefit = .412(7,800) = 3213.60

$$\text{Retired AL} = 3213.60\,\ddot{a}_{55}^{(12)}$$

$$= 34{,}643 = 3213.60(10.78)$$

$$\Delta = 8{,}889 = 43{,}532 - 34{,}643$$

Gain since active AL > retiree AL

A

Spring 1996 EA-1B

4 The statement about the Aggregate cost method for 1995 is not used in this solution. You must do Entry age normal accrued liability calculations at 1-1-96 for both Smith and Brown to allocate the assets under the Individual aggregate method.

The quickest calculation of the EAN accrued liability is

$$EAN\ AL = PVB_{CA}\left(\frac{{}^{s}\ddot{a}_{EA:\overline{CA-EA}|}}{{}^{s}\ddot{a}_{EA:\overline{RA-EA}|}}\right) \text{ for pay related benefits}$$

In this problem, benefits aren't pay related, so we'll use annuity values with no salary scale:

	Smith	Brown
1-1-96 Age	40	55
Hire age	35	40
Past service	5	15
Total service	30	25
Projected benefit	$7200 = 12(\$20)(30)$	$6000 = 12(\$20)(25)$
PVB_{CA}	$7200\,\ddot{a}_{65}^{(12)}\,D_{65}/D_{40}$	$6000\,\ddot{a}_{65}^{(12)}\,D_{65}/D_{55}$
	$= 13266 = 7200(10)(1.07)^{-25}$	$30501 = 6000(10)(1.07)^{-10}$
EAN_{AL}	$13,266\,(\ddot{a}_{\overline{5}|.07}/\ddot{a}_{\overline{30}|.07})$	$30,501\,(\ddot{a}_{\overline{15}|.07}/\ddot{a}_{\overline{25}|.07})$
	$= 4,383$	$= 23,838$ $\quad \Sigma = 28,22$

No longer need to do any calculations for Smith

	Smith	Brown
Allocated AAV	$20,000\left(\frac{4,383}{28,221}\right)$	$20,000\left(\frac{23,838}{28,221}\right)$
		$= 16,894$
PV future benefits		30,501 from above
PVNC = PVB - Alloc AAV		$13,607 = 30,501 - 16,894$
Annuity for normal cost		$\ddot{a}_{\overline{10}|.07} = 7.5152$
Normal cost		$1811 = \frac{13,607}{7.5152}$

C

Spring 1996 EA-1B

5 Under the Individual level premium cost method, there are new layers of normal cost for each change in the projected benefit. You will need to calculate the layers created at plan inception (1/1/87) and at the 1/1/96 plan change. Since ILP is an individual cost method, you do not use the assets of 5,000 in calculating the normal cost.

$$\Delta \text{ILP NC} = \frac{(\Delta \text{Projected benefit})\, \ddot{a}^{(12)}_{RA}\, D_{RA}/D_{CA}}{{}^{s}\ddot{a}_{CA:\overline{RA-CA|}}}$$

Since this plan benefit is not pay related, you will calculate the annuity with no salary scale

	1-1-87	1-1-96
Age	30	39
Total service	40	40
Monthly Projected benefit	600	720
Δ Projected benefit	600	120
Δ PV future benefits	$600(12\ddot{a}^{(12)}_{65})D_{65}/D_{30}$	$120(12\ddot{a}^{(12)}_{65})D_{65}/D_{39}$
	$=600(104.83)(1.07)^{-35}$	$=120(104.83)(1.07)^{-26}$
Δ ILP NC	$\dfrac{600(104.83)(1.07)^{-35}}{\ddot{a}_{\overline{35}\rvert .07}}$	$\dfrac{120(104.83)(1.07)^{-26}}{\ddot{a}_{\overline{26}\rvert .07}}$
	$=600(104.83)/\ddot{s}_{\overline{35}\rvert .07}$	$=120(104.83)/\ddot{s}_{\overline{26}\rvert .07}$
	$=425.24$	$=171.19$

1/1/96 ILP NC = 425.24 + 171.19 = 596.43

C

Spring 1996 EA-1B

6 Since Attained age normal is an aggregate cost method, and 1/1/96 is the plan's effective date, the 1/1/96 UAL equals the unit credit accrued liability at 1/1/96 less the initial assets (assumed to be zero).

Under Unit credit, the accrued liability is defined as the present value of the accrued benefit:

1/1/96 Age 45 Hire age 35

past service 10 Total service 30

Accrued benefit \$10(12)(10) = 1200 Projected benefit \$10(12)(30) = 3600

$$UCAL = 1200\, \ddot{a}_{65}^{(12)}\, D_{65}/D_{45}$$
$$= 1200\left[8.74\,(1.07)^{-20}\right] \quad \text{no pre-retirement decrement}$$
$$= 2{,}710 \qquad = 1200(2.2586)$$

Under Entry age normal, the accrued liability can be calculated using this formula

$$EAN\ AL = (PVB_{CA})\left(\frac{{}^{s}\ddot{a}_{EA:\overline{CA-EA}|}}{{}^{s}\ddot{a}_{EA:\overline{RA-EA}|}}\right) \quad \text{for pay related benefi}$$

You should use annuities with no salary scale, since the benefits are not related to pay:

$$EAN\ AL = \left(3600\, \ddot{a}_{65}^{(12)}\, D_{65}/D_{45}\right)\left(\frac{\ddot{a}_{35:\overline{10}|}}{\ddot{a}_{35:\overline{30}|}}\right)$$
$$= 3600(2.2586)\left(\ddot{a}_{\overline{10}|.07} / \ddot{a}_{\overline{30}|.07}\right)$$
$$= 4{,}602$$

$\Delta = 4602 - 2710 = 1892$ D

Spring 1996 EA-1B

7 This problem requires you to do two valuations, one with Smith alive, and one treating Smith as having died. The tricky aspect of the problem is the ten year certain and life normal form. When Smith dies, there is still an eight year certain annuity to pay.

Under the Aggregate method, the normal cost is calculated as the PVNC divided by an average temporary annuity. In this problem, the annuity is solely based on Brown, who is the only active employee. The PVNC is equal to PV of future benefits minus the asset value of 110,000.

	Smith	Brown
1-1-96 age	67	45
Retirement age	65	65
Total svc at retirement	20	30
Projected benefit	$\$12{,}000 = \$50(12)(20)$	$\$18{,}000 = \$50(12)(30)$
Present value factor	$\ddot{a}^{(12)}_{\overline{8}\rvert} + {}_{8\rvert}\ddot{a}^{(12)}_{67}$	$\left(\ddot{a}^{(12)}_{\overline{10}\rvert} + {}_{10\rvert}\ddot{a}^{(12)}_{65}\right) D_{65}/D_{45}$
	$= 6.1954 + N^{(12)}_{75}/D_{67}$	$= \left(7.2871 + N^{(12)}_{75}/D_{65}\right)(1.07)^{-20}$
	$= 6.1954 + \frac{217{,}236}{78{,}601}$	$= \left(7.2871 + \frac{217{,}236}{94{,}414}\right) .2584$
	$= 6.1954 + 2.7638$	$= 2.4777$

Keep Smith's present value separated between the certain only portion - this allows easy identification of the change in normal cost. The change in PVNC due to Smith's death is $12000(2.7638) = 33{,}166$. The annuity factor in both valuations is $\ddot{a}_{20\rvert 07}$, which is for Brown. The change in normal cost is $33166/\ddot{a}_{20\rvert 07} = 2926$ E

(7) If you want to see all the rest of the calculations, here they are:

Valuation for Smith alive	Smith	Brown	
PV future benefits	$12000(6.1954+2.7638)$ $=74,344+33,165$	$18000(2.4777)$ $=44,599$	$\Sigma=152,109$
PVNC = PVB − AAV	$=152,109-110,000$ $=42,109$		
Avg annuity	N/A	$\ddot{a}_{45:\overline{20}\rceil}$ $=\ddot{a}_{\overline{20}\rceil .07}$ (no decrements)	
normal cost	$42,109/11.3356$ $=3,715$		

Valuation for Smith dead	Smith	Brown	
PV future benefits	$12,000(6.1954)$ $=74,344$	$18,000(2.4777)$ $=44,599$	$\Sigma=118,943$
PVNC = PVB − AAV	$=118,943-110,000$ $=8,943$		
Avg annuity	N/A	$\ddot{a}_{45:\overline{20}\rceil}$ $=\ddot{a}_{\overline{20}\rceil .07}$ (no decrements)	
Normal cost	$8,943/11.3356$ $=788$		

$\Delta NC = 3715-788 = 2926$

E

Spring 1996 EA-1B

Solution corrected 3/8/99

8 This is a relatively straightforward actuarial equivalence problem. You should write down the formulas for the present value of benefits under Option A and B. After simplifying them, you can set them equal to each other and solve for X:

Option A: $12[(1000+X)\ddot{a}^{(12)}_{60:\overline{5}|} \; X(5|\ddot{a}^{(12)}_{60})]$

$12[1000\ddot{a}^{(12)}_{60:\overline{5}|} + X\ddot{a}^{(12)}_{60})]$

$12[1000(\ddot{a}^{(12)}_{60} - \frac{D_{65}}{D_{60}}\ddot{a}^{(12)}_{65}) + X\ddot{a}^{(12)}_{60}]$

$12[1000(9.815 - \frac{94{,}414}{144{,}405}(8.736)) + X(9.815)]$

$12[4103 + 9.815X]$

Option B: $12[(1000-X)\ddot{a}^{(12)}_{60} + (1000-X)\ddot{a}^{(12)}_{60} + (2X-1000)\ddot{a}^{(12)}_{60:60}]$

This expression is derived by writing down the first two 1000-x terms as the annuities payable to either survivor. When all the annuitants are alive, the total payment should be 1000, which equals $2(1000-X) - (2X-1000)$

$12[(2000-2X)\ddot{a}^{(12)}_{60} + (2X-1000)\ddot{a}^{(12)}_{60:60}]$

$12[1000(2\ddot{a}^{(12)}_{60} - \ddot{a}^{(12)}_{60:60}) + X(2\ddot{a}^{(12)}_{60:60} - 2\ddot{a}^{(12)}_{60})]$

$12[1000(2(9.815)-8.094) + X(2)(8.094-9.815)]$

$12[11536 - 3.442X]$

$\therefore \quad 4{,}103 + 9.815X = 11{,}536 - 3.442X$

$13.257X = 7{,}433$

$X = 560.68$

B

Spring 1996 EA-1B

9 There are two ways to work this gain and loss problem. One requires you to solve for probabilities of survival, and to write down a specific formula for the expected accrued liability for each participant.

The easier solution uses the general formula for non-investment gains and losses:

non-inv G/L = $eAL_1 - AL_1$

$eAL_1 = (1+i)(NC_0 + AL_0) - (\text{actual BP} + \text{interest})$

$\uparrow$ assumed zero for retirees

For each retiree, AL_0 is simply the 1-1-95 PVB

The actual BP + interest is $[1 + \frac{13}{24}(.07)](12)(\text{monthly benefit})$ for each retiree, since they all received 12 monthly benefits.

	Smith	Brown	Green	
1-1-95 age	60	65	70	
Annual benefit	48,000	60,000	72,000	
AL_0 = 1-1-95 PVB	$48{,}000\ddot{a}_{60}^{(12)}$	$60{,}000\ddot{a}_{65}^{(12)}$	$72{,}000\ddot{a}_{70}^{(12)}$	
	$=470{,}880$	$=524{,}400$	$=547{,}200$	$\Sigma = 1{,}542{,}48$

$eAL_1 = 1.07(1{,}542{,}480) - (1 + \frac{13}{24}(.07))(48{,}000 + 60{,}000 + 72{,}000)$

$= 1.07(1{,}542{,}480) - 1.0379((180{,}000)$

$= 1{,}463{,}629$

	Smith	Brown	Green	
1-1-96 age	61	66	71	
Status	alive	dead	alive	
AL_1 = 1-1-96 PVB	$48{,}000\ddot{a}_{61}^{(12)}$	zero	$72{,}000\ddot{a}_{71}^{(12)}$	
	$=460{,}800$	zero	$=530{,}640$	$\Sigma = 991{,}44$

G/L = 1,463,629 − 991,440 = 472,189 gain

B

(9) continued

The harder way to work this problem is based on knowing formulas for the eAL₁ for a monthly life annuity: $eAL_1 = (p_x \ddot{a}^{(12)}_{x+1} - (11/24) q_x)(\text{Annual benefit})$

You must derive the values for p_x for each retiree, based on this formula:

$$v p_x \ddot{a}_{x+1} = \ddot{a}_x - 1.0 \Rightarrow p_x = (1+i)(\ddot{a}_x - 1.0)/\ddot{a}_{x+1}$$

Since you are given monthly annuities, you have

$$p_x = (1+i)\left(\ddot{a}^{(12)}_x - \tfrac{13}{24}\right) / \left(\ddot{a}^{(12)}_{x+1} + \tfrac{11}{24}\right)$$

	Smith	Brown	Green
1-1-95 age x	60	65	70
p_x	$\frac{1.07(\ddot{a}^{(12)}_{60} - \frac{13}{24})}{\ddot{a}^{(12)}_{61} + \frac{11}{24}}$	$\frac{1.07(\ddot{a}^{(12)}_{65} - \frac{13}{24})}{\ddot{a}^{(12)}_{66} + \frac{11}{24}}$	$\frac{1.07(\ddot{a}^{(12)}_{70} - \frac{13}{24})}{\ddot{a}^{(12)}_{71} + \frac{11}{24}}$
	$= .9860$	$= .9781$	$= .9640$
factor for eAL_1	$p_{60}\ddot{a}^{(12)}_{61} - \frac{11}{24}q_{60}$	$p_{65}\ddot{a}^{(12)}_{66} - \frac{11}{24}q_{65}$	$p_{70}\ddot{a}^{(12)}_{71} - \frac{11}{24}q_{70}$
	$= 9.4588$	$= 8.3139$	$= 7.0941$
eAL_1	48,000(9.4588)	60,000(8.3139)	72,000(7.0941)
	= 454,022	= 498,833	= 510,775 Σ=1,463,6
AL_1	$48{,}000\,\ddot{a}^{(12)}_{61}$	zero	$72{,}000\,\ddot{a}^{(12)}_{71}$
	= 460,800	zero	= 530,640 Σ=991,44

G/L = 1,463,630 − 991,440 = 472,190 gain

B

Spring 1996 EA-1B

10 You must calculate the normal cost under both the Entry age normal method and the Unit credit method. Under the EAN method, the normal cost is defined as $PVB_{EA} / {}^{s}\ddot{a}_{EA:\overline{RA-EA}|}$. Since this plan's benefits are not pay related, you will use an annuity without salary scale.

1-1-96 Age 60   Total service 45
Hire age 20   Past service 40

$$EANC = PVB_{EA} / \ddot{a}_{EA:\overline{RA-EA}|}$$

$$\text{Proj Benefit} = \$20(12)(25)$$
$$= 6{,}000$$

$$EANC = \frac{6000(\ddot{a}^{(12)}_{65})D_{65}/D_{20}}{\ddot{a}_{20:\overline{45}|}}$$

$$= 6000(9.24)(1.07)^{-45} / \ddot{a}_{\overline{45}|.07}$$ no pre-ret decrements

$$= 6000(9.24) / \ddot{s}_{\overline{45}|.07}$$

$$= 181.32$$

The unit credit normal cost is normally the present value of the next year's benefit accrual. Since this participant has more than 25 years of service, their accrued benefit is \$20(12)(25), or 6000. There are no benefit accruals beyond 25 years, so the normal cost is zero under Unit credit!

$$\Delta NC = 0 - 181 = 181 \text{ (absolute value)}$$

A

Spring 1996 EA-1B

11 This is a typical Frozen initial liability problem. Since 1/1/95 is the initial effective date, you must do an Entry age normal valuation at that date, and establish the initial FIL UAL = EAN AL. Then you must calculate the FIL normal cost, and write down the UAL from 1/1/95 to 1/1/96.

1-1-95 Age 49 Total service 45

Hire age 20 Past service 29

Projected benefit = \$50(12)(45)

= 27,000

$$EAN\ AL = PVB_{CA}\left({}^{s}\ddot{a}_{EA:\overline{CA-EA}|} \,/\, {}^{s}\ddot{a}_{EA:\overline{RA-EA}|}\right)$$

Since benefit are not pay related, use annuities with no salary scales. Also can simplify due to no pre-ret decrements:

$$EAN\ AL = 27{,}000\,\ddot{a}_{65}^{(12)}\frac{D_{65}}{D_{49}}\left(\frac{\ddot{a}_{20:\overline{29}|}}{\ddot{a}_{20:\overline{45}|}}\right)$$

$$= 27{,}000(9.24)(1.07)^{-16}\left(\ddot{a}_{\overline{29}|.07}/\ddot{a}_{\overline{45}|.07}\right)$$

$$= 76{,}260 = FIL\ UAL$$

Since this is the initial year of the plan, and you have a single participant, the EAN NC = FIL NC:

$$EAN\ NC = 27{,}000\,\ddot{a}_{65}^{(12)}(D_{65}/D_{20})/\ddot{a}_{20:\overline{45}|}$$

$$= 27{,}000(9.24)(1.07)^{-45}/\ddot{a}_{\overline{45}|.07}$$

$$= 816 = FIL\ NC$$

$$1\text{-}1\text{-}96\ eUAL = (1+i)(NC_0 + UAL_0) - (\text{Contrib} + \text{interest})$$

$$= 1.07(816 + 76{,}260) - 1.07(5{,}000)$$

$$= 77{,}121$$

C

(11) continued

If you calculated the 1-1-95 FIL normal cost the long way, it does equal 816:

$$\begin{aligned}
\text{1-1-95 PVB} &= 27{,}000(9.24)(1.07)^{-16} \\
&= 84{,}508 \\
\text{PVNC} &= \text{PVB} - (\text{AAV} + \text{UAL}) \\
&= 84{,}508 - 76{,}260 \\
&= 8{,}248 \\
\text{FIL NC} &= \text{PVNC} / (\text{average annuity}) \\
&= 8{,}248 / \ddot{a}_{49:\overline{16}|} \\
&= 8{,}248 / \ddot{a}_{\overline{16}|.07} \\
&= 816
\end{aligned}$$

This will equal the EAN normal cost when you are in the first year of the plan, and your population's average temporary annuity under FIL equals the (EAN PVNC / EAN NC). This will be true for a plan population of one, or for a population where all participants are clones: same age and service, etc.

$$\begin{aligned}
\text{FIL PVFB} &= \text{EAN PVFB} \\
\text{FIL UAL} &= \text{EAN AL} = \text{EAN PVB} - \text{EAN PVNC} \\
\text{FIL PVNC} &= \text{FIL PVB} - (\text{FIL UAL} + \text{FIL AV}) \\
&= \text{EAN PVNC} \\
\text{FIL NC} &= \text{EAN PVNC} / (\text{average annuity})
\end{aligned}$$

Spring 1996 EA-1B

12 This is a potentially tricky PUC problem. You should ask why they defined the benefit accrual based on monthly percentages. The answer is that the participant does not have an integer number of years of service at 1/1/96, which is fairly unique for this exam. The trick is that the service is 9.25 years at 1/1/96, so the benefit accrual in the next year is based on both the 2/12 % and the 1/12 % accrual rates

1-1-96 Age 53
Service 9.25

Age 53 pay = 50,000
Age 64 pay = 76,973 = $50{,}000(1.04)^{11}$
$FAE\overline{3}|_{64} = 74{,}050 = 76{,}973(\ddot{a}_{\overline{3}|.04})/3$

Under Projected unit credit, the normal cost is defined as the present value of the change in the "funding accrued benefit". The funding accrued benefit can be calculated by applying the benefit formula to past service at 1-1-96 and 1-1-97 to the projected FAE3 at age 64:

	1-1-96	1-1-97
Service	9.25	10.25
Benefit accrual	.02 (9.25) = .1850	.02 (10) + .01 (.25) = .2025
Funding accrued ben	.1850 (74,050)	.2025 (74,050)
Δ FAB	.0175 (74,050) = 1296	

Normal cost = $1296\, \ddot{a}_{65}^{(12)}\, D_{65}/D_{53} = 1296(8.74)(1.07)^{-12} = 5029$ (D)

Spring 1996 EA-1B

13 This is a rare problem on the one year term cost concept. The total cost for the plan would be the normal cost under Individual level premium, excluding the death benefits, plus the one year term cost for the death benefit. The one year term cost is calculated as

$$v \cdot q_x \cdot (\text{death benefit})$$

The cost of the plan on a split funded basis should be the Aggregate normal cost plus the insurance premium for the death benefit.

ILP valuation

1-1-96 Age 40 (age at plan inception = 40)

$$\text{level dollar } \Delta \text{ILP NC} = \frac{(\Delta \text{Projected benefit})\, \ddot{a}^{(12)}_{RA}\, D_{RA}/D_x}{\ddot{a}_{x:\overline{RA-x}|}}$$

$$= 12000(8.74)\,D_{65}/(N_{40}-N_{65})$$

$$= 12000(8.74)\,94{,}414/(8{,}452{,}729-868{,}052)$$

$$= 1{,}306$$

One year term cost

$$= v\,q_{40}\,(100{,}000)$$

$$= (C_{40}/D_{40})\,100{,}000$$

$$= [(M_{40}-M_{41})/D_{40}]\,100{,}000$$

$$= 199$$

Total cost under method A = 1306 + 199 = 1505

(13) continued

AGG valuation

For a split funded plan, use the cash surrender value at age 65 to partially fund the retirement benefits. Then apply the Aggregate method to the remaining benefit, and determine the normal cost. The resulting side fund normal cost should be added to the insurance premium to calculate the total cost under method B.

CSV at 65	41,900	
Insurance provided ben	4,794	$= 41{,}900/8.74$
Net benefit	7,206	$= 12{,}000 - 4{,}794$
PV future benefits		$= 7206\,\ddot{a}_{65}^{(12)}\,D_{65}/D_{40}$
	9,404	$= 7206(8.74)\frac{94{,}414}{632{,}275}$
PVNC = PVB − AAV	9,404	
average annuity		$= \frac{N_{40} - N_{65}}{D_{40}}$
	11.9959	$= \frac{8{,}452{,}729 - 868{,}052}{632{,}275}$
AGG NC	784	$= 9404/11.9959$
Insurance premium	1,585	
Total cost - method B	2,369	$= 1585 + 784$

Δ in costs $= 2369 - 1505 = 865$

C

Spring 1996 EA-1B

14 With the withdrawal decrement, the annuities are complex to calculate. You can determine the accrued liability on either a prospective or a retrospective basis. The preferred approach is calculation under the retrospective basis, and to avoid calculating the accumulation annuity! It is clearer to simply accumulate the prior year's accrued liability one year at a time:

1-1-96 age 38
entry age 35

$$AL_1 = \frac{D_x}{D_{x+1}}(NC_0 + AL_0)$$

$$= \frac{(1+i)}{p_x^{(T)}}(NC_0 + AL_0)$$

Age x	$p_x^{(T)}$	AL_x	NC_x	$AL_{x+1} = (1.08/p_x^{(T)})(AL_x + NC_x)$
35	.5	0	10,000	21,600 = (1.08/.50)(10,000)
36	.6	21,600	10,000	56,880 = (1.08/.60)(21,600+10,000)
37	.7	56,880	10,000	103,186 = (1.08/.70)(56,880+10,000)
38		103,186		

E

One aspect of this result that is confusing is the magnitude of the accrued liability - it is extremely large!

Corrected 2/12/2000

Spring 1996 EA-1B

15 There are two approaches for solving this gain and loss problem. One relies on knowing the formula for the expected liability for an annual life annuity benefit. The other way relies on the general formula for a non-investment gain and loss:

$$\text{non-inv } G/L = eAL_1 - AL_1$$

$$eAL_1 = (1+i)(NC_0 + AL_0) - (\text{actual BP} + \text{interest})$$

NC_0 → assume zero for retirees

One trick to working this problem under either approach is that you must derive the value of $\ddot{a}_{x+1:y+1}$, which requires you to solve for p_x and p_y:

$$1 + v p_x \ddot{a}_{x+1} = \ddot{a}_x \,, \qquad 1 + v p_{xy} \ddot{a}_{x+1:y+1} = \ddot{a}_{xy}$$

$$p_x = \left(\frac{\ddot{a}_x - 1.0}{\ddot{a}_{x+1}}\right)(1+i) \qquad \ddot{a}_{x+1:y+1} = (\ddot{a}_{xy} - 1.0)\left(\frac{1+i}{p_x p_y}\right)$$

$$p_y = \frac{9.301(1.07)}{10.059} = .9894 \qquad p_x = \frac{7.157(1.07)}{7.915} = .9675$$

$$= (6.281)\left(\frac{1.07}{.9894(.9675)}\right) = 7.0209$$

Now you can calculate the value of AL_1

$$AL_1 = 10{,}000\left(\ddot{a}_{x+1} + .5\left(\ddot{a}_{y+1} - \ddot{a}_{x+1:y+1}\right)\right)$$
$$= 10{,}000\left(7.915 + .5(10.059 - 7.0209)\right)$$
$$= 94{,}341$$

$$eAL_1 = 1.07(10{,}000)\left(\ddot{a}_x + .5(\ddot{a}_y - \ddot{a}_{xy})\right) - 1.07(10{,}000)$$
$$= 1.07(10{,}000)\left(8.157 + .5(10.301 - 7.281) - 1.0\right)$$
$$= 92{,}737$$

Δ = loss of 1604 = 94,341 − 92,737 (E)

(15) continued

You can also calculate eAL_1 by deriving the liabilities under the four possible cases, and applying the appropriate probabilities. This method still requires you to derive value for p_x, p_y, and $\ddot{a}_{x+1:y+1}$ as shown on the prior page

Case	Probability	Liability	Probability x Liability
x only alive	$p_x(1-p_y) = .9675(.0106)$	$10{,}000\,\ddot{a}_{x+1}$	812
y only alive	$p_y(1-p_x) = .9894(.0325)$	$5{,}000\,\ddot{a}_{y+1}$	1,617
both alive	$p_x p_y = .9675(.9894)$	$10{,}000(\ddot{a}_{x+1} + .5(\ddot{a}_{y+1} - \ddot{a}_{x+1:y+1}))$	90,307
neither alive	$(1-p_x)(1-p_y) = .0106(.0325)$	θ	θ

Expected liability = Σ 92,736

$$\text{Loss} = AL_1 - eAL_1$$

= 94,341 prior page − 92,736

= 1,605

(E)

Spring 1996 EA-1B

16 In this problem you are given the Entry age normal cost on the old plan. You can calculate the new EAN normal cost on the new plan by comparing the projected benefit under both plans. You must calculate the salary weighted annuity to determine the EAN accrued liability on a retrospective basis:

1-1-96 Age 46   1-1-88 EANC = 6500

Entry age 38   1-1-96 EANC = 9603 = $6500(1.05)^8$

Old plan AL = $9603\,({}^{s}\ddot{s}_{38:\overline{8|}})$

show EANC "payments" $9603(1.05)^{-8}$ ... $9603(1.05)^{-1}$

38   39   40 ... 45   46

$$AL = 9{,}603\left[\left(\frac{1.07}{1.05}\right)^8 + \dots + \left(\frac{1.07}{1.05}\right)^2 + \left(\frac{1.07}{1.05}\right)\right]$$

$= 9{,}603\,\ddot{s}_{\overline{8|}j}$ where $1+j = \frac{1.07}{1.05} = 1.0190$

$= 83{,}714$

	Old Plan	New Plan
Assumed Age 64 Pay	1.0	1.0
Final average period	5	3
$FAE_{\overline{n}|64}$	$\ddot{a}_{\overline{5|}.05}/5$ = .9092	$\ddot{a}_{\overline{3|}.05}/3$ = .9531
Benefit accrual %	1.25%	1.75%
Projected benefit	1.1365%(svc)	1.6680%(svc)

New plan accrued liability = $\left(\frac{1.6680}{1.1365}\right)83{,}714$

Δ accrued liability = $(.4677)83{,}714$

$= 39{,}151$ D

Spring 1996 EA-1B

17 This is a fairly representative valuation problem, which is designed to test your handling of the non-active participants. You must calculate the projected benefit for Smith, and use the accrued benefits for the other two participants. Smith is the only employee considered in calculating the average temporary annuity.

Smith 1-1-96 age 35 Past service 10 Total service 40

age 35 pay 48,000

age 64 pay 197,575 = $48,000(1.05)^{29}$

Projected benefit 79,030 = .01(40)(197,575)

Slight trick - ignore §401(a)(17) limit for this exam!

	Smith	Brown	Green	Σ
Age	35	50	65	
Annual benefit	79,030	2,400	3,600	
Present value factor	$_{30\vert}\ddot{a}_{35}^{(12)}$ = 1.0	$_{15\vert}\ddot{a}_{50}^{(12)}$ = 4.0	$\ddot{a}_{65}^{(12)}$ = 10.0	
PVB	79,030	9,600	36,000	124,630
PVNC	64,630 = PVB - AAV = 124,630 - 60,000			
Average annuity	$^{s}\ddot{a}_{35:\overline{30}\vert}$ = 12.5			

Aggregate normal cost 5,170 = $\dfrac{64,630}{12.5}$

B

Spring 1996 EA-1B

18 This is one of the trickiest Projected unit credit questions ever asked on the exam. When you start working the problem and simplifying results, it is not clear that you can actually calculate an answer!

You must allow for the value of future vesting benefits when you calculate the accrued liability. With a pay-related benefit, the funding accrued benefit must be calculated reflecting the different projected pay at each exit age.

I'll analyze the "old assumptions" PUC accrued liability and get it in simplest terms first:

$$
\begin{aligned}
PUC\,AL = {} & 2\%(\text{past svc})(FAE3|_{64})(D_{65}^{(T)}/D_x^{(T)})\,\ddot{a}_{65}^{(12)} \\
& + 2\%(\text{past svc})(FAE3|_{49})(D_{50}^{(T)}/D_x^{(T)}).4({}_{15|}\ddot{a}_{50}^{(12)}) \\
& + 2\%(\text{past svc})(FAE3|_{54})(D_{55}^{(T)}/D_x^{(T)}).25({}_{10|}\ddot{a}_{55}^{(12)}) \\
& + 2\%(\text{past svc})(FAE3|_{59})(D_{60}^{(T)}/D_x^{(T)}).20({}_{5|}\ddot{a}_{60}^{(12)})
\end{aligned}
$$

Now replace "pastsvc" with PS, and factor out various common terms, plus break down the D/D terms:

$$PUC\,AL = .02(PS)(\ddot{a}_{\overline{3}|.04}/3)\,\ddot{a}_{65}^{(12)} \text{ times}$$

$$
\begin{aligned}
[\text{Ret at 65}: \quad & (1.04)^{64-x}\; v^{65-x}\; {}_{65-x}p_x^{(T)} \\
+ \text{Withd at 50}: \quad & (1.04)^{49-x}\; v^{50-x}\; {}_{50-x}p_x^{(T)}\,(.40)\,v^{15} \\
+ \text{Withd at 55}: \quad & (1.04)^{54-x}\; v^{55-x}\; {}_{55-x}p_x^{(T)}\,(.25)\,v^{10} \\
+ \text{Withd at 60}: \quad & (1.04)^{59-x}\; v^{60-x}\; {}_{60-x}p_x^{(T)}\,(.25)\,v^{5}\;]
\end{aligned}
$$

(18) continued

Finally, factor out even more items:

Old assumptions

$PUC\ AL = .02(PS)(\ddot{a}_{\overline{3}|.04}/3)v^{65-x}\ddot{a}_{65}^{(12)}(1.04)^{49-x}$ times

$$\begin{aligned}
&[\text{Ret at } 65: (1.04)^{15}(.60)(.75)(.80) &&= .6483\\
&+ \text{Withd at } 50: (1.04)^{0}(.40) &&= .4000\\
&+ \text{Withd at } 55: (1.04)^{5}(.60)(.25) &&= .1825\\
&+ \text{Withd at } 60: (1.04)^{10}(.60)(.75)(.20) &&= .1332]\\
&= \text{common terms } [1.3640]
\end{aligned}$$

New assumptions

$PUC\ AL = .02(PS)(\ddot{a}_{\overline{3}|.04}/3)v^{65-x}\ddot{a}_{65}^{(12)}(1.04)^{49-x}$ times

$$\begin{aligned}
&[\text{Ret at } 65: (1.04)^{15}(.50)(.80) &&= .7204\\
&+ \text{Withd at } 50: (1.04)^{0}(.50) &&= .5000\\
&+ \text{Withd at } 55: (1.04)^{5}(.50)(.20) &&= .1217]\\
&= \text{common terms } [1.3420]
\end{aligned}$$

$$\text{Ratio} = \frac{1.3420}{1.3640} = .9839$$

A

Spring 1996 EA-1B

19) This is a combination of two problem types usually on the exam, employee contributions and projected valuations. In the projected valuation problems, the key is determination of the average temporary annuity. This problem only tests that you know how to calculate the Aggregate PVNC correctly with employee contributions:

Agg PVNC = PVB - AAV - PVEEC

To calculate the average annuity at 1-1-96, you must determine some items at 1-1-95:

1-1-95 PVNC = 100,000 - 25,000 - 10,000

= 65,000

NC = 7,200

average annuity = 9.0278 = 65,000 / 7,200

If all assumptions regarding decrements are met, and there are no new entrants or retirements, you can calculate the average annuity the next year:

$1 + v p_x \ddot{a}_{x+1} = \ddot{a}_x$ $\qquad$ $1 + v p_x \ddot{a}_{x+1:\overline{n-1}|} = \ddot{a}_{x:\overline{n}|}$

$\ddot{a}_{x+1:\overline{n-1}|} = (\ddot{a}_{x:\overline{n}|} - 1.0)\left(\dfrac{1+i}{p_x}\right)$

$= 8.0278\,(1.07)$

$= 8.5897$

1-1-96 PVNC = 60,500 = 1.07(100,000) - 24,500 - 22,000

NC = 7,043 = 60,500 / 8.5897

Note - with no decrements, PVB increases by interest each year (B)

Spring 1996 EA-1B

20 When calculating a salary scale G/L, simply compare the accrued liability on expected pay to the accrued liability on actual pay. With a pay related plan, you have some messy calculations under the Entry age normal method. The quickest calculation of the EAN accrued liability is

$$\text{EAN AL} = (PVB_{CA})\left(\frac{{}^{s}\ddot{a}_{EA:\overline{CA-EA}|}}{{}^{s}\ddot{a}_{EA:\overline{RA-EA}|}}\right)$$

1-1-96 age 46   past service 16   Age 46 pay 50,000

entry age 30   ~~total~~ service 35   Age 64 pay $120{,}331 = 50{,}000(1.05$

$FAE\ 3/64\ \ 114{,}692 = 120{,}331\left(\frac{\ddot{a}_{\overline{3}|}}{3}\right.$

Proj Benefit $68{,}815 = .6(114{,}692)$

$$\text{actual AL} = 68{,}815\ \ddot{a}^{(12)}_{65}(D_{65}/D_{46})\left({}^{s}\ddot{a}_{30:\overline{16}|} / {}^{s}\ddot{a}_{30:\overline{35}|}\right)$$

You need to write out the series for one of the pay weighted annuities to calculate it directly:

$${}^{s}\ddot{a}_{30:\overline{16}|} = 1 + \left(\frac{1.05}{1.07}\right) + \dots + \left(\frac{1.05}{1.07}\right)^{15}$$

$$= \ddot{a}_{\overline{16}|j} \quad \text{where } 1+j = \frac{1.07}{1.05} = 1.0190$$

$$\text{actual AL} = 68{,}815(8.736)(1.07)^{-19}\left(\ddot{a}_{\overline{16}|1.90\%} / \ddot{a}_{\overline{35}|1.90\%}\right)$$

$$= 68{,}815(8.736)(1.07)^{-19}(13.9413/25.8594)$$

$$= 89{,}617$$

expected AL is based on expected pay, which would be 1.05(50,000). This produces a projected benefit of 1.05(68,815), and so on

$$eAL_1 - AL_1 = 1.05(89{,}617) - 89{,}617$$

$$= .05(89{,}617) = 4481$$

B