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SPRING 1994 EA-1B EXAM SOLUTIONS

Revision History:	
02/12/00	Corrected problem 01 – improved solution
02/12/00	Corrected problem 07 – added clearer solution
02/12/00	Corrected problem 16 - description of earnings definition

Revised 2/12/2000

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1 Frozen Initial Liability is an aggregate cost method. The initial accrued liability at 1-1-94 is derived using the Entry Age Normal method. The problem clarifies that "entry age" should be based on actual hire age (not 1-1-94).

The key point of this problem is careful handling of the different interest rates before and after retirement. Use the Unit Credit accrued liability to derive the $\ddot{a}_{65}^{(12)}$ at 7% interest. Then you can calculate the EAN accrued liability and the FIL normal cost. Be careful to use 8% interest pre-ret!

1-1-94 Age 60 Past service 10 Accrued benefit 2,400
hire age 50 total service 15 Projected benefit 3,600

$$U.C.\ AL = PV(\text{accrued ben}) = 2{,}400 \frac{D_{65}}{D_{60}} \ddot{a}_{65}^{(12)} \quad \Big\} \text{ all factors at 7\%}$$

$$14.900 = 2400(1.07)^{-5} \ddot{a}_{65}^{(12)} \quad \text{no pre-ret decrements}$$

$$\ddot{a}_{65}^{(12)} \text{ at 7\%} = (14.900/2{,}400)(1.07)^5 = 8.7075$$

$$E.A.N.\ AL = EANC(N_{50} - N_{60})/D_{60} \quad \text{on a retrospective basis}$$

$$EANC = PVB_{EA} / \ddot{a}_{EA:\overline{RA-EA|}} \quad \text{by definition under E.A.N.}$$

$$PVB_{EA} = 3600(\ddot{a}_{65}^{(12)} \text{ at 7\%})(D_{65}/D_{50})$$

$$\ddot{a}_{EA:\overline{RA-EA|}} = (N_{50} - N_{65})/D_{50}$$

all factors except $\ddot{a}_{65}^{(12)}$ are at 8%

$$EANC = \frac{3600(8.7075)}{(N_{50}-N_{65})/D_{65}} = \frac{3600(8.7075)}{\ddot{s}_{\overline{15|}}\ 1.08} \quad \text{no pre-ret decr}$$

$$= 1{,}069$$

Since this is the initial plan year, and you only have one participant, the FIL normal cost is also 1,069! (A)

(1) Continued

If you did not know the FIL NC would equal the EAN NC in the first year, you can calculate the result "the longway":

EAN AL = EANC($\ddot{s}_{50:\overline{10|}}$) = 1069 ($\ddot{s}_{\overline{10|}.08}$) = 16,725

1-1-94 FIL UAL = EAN AL − AAV = 16,725

FIL PVNC = PVB − UAL − AAV

PVB = 3600 ($\ddot{a}_{65}^{(12)}$ at 7%) D_{65}/D_{60}

= 3600 (8.7075)$(1.08)^{-5}$ = 21,334

PVNC = 21,334 − 16,725 − 0 = 4,610

$$\text{FIL NC} = \frac{\text{PVNC}}{\text{avg } \ddot{a}_{x:\overline{RA-x|}}} = \frac{4,610}{\ddot{a}_{\overline{5|}.08}} = 1,069$$

The FIL NC will equal the EAN NC when you are in the first year of a plan, and the plan's average temporary annuity under FIL is equal to (EAN PVNC/EAN NC). This will be true for a plan population of one, or a population of clones, all of whom have the same age and entry age:

FIL PVFB = EAN PVFB

FIL UAL = EAN AL = EAN PVB − EAN PVNC

FIL PVNC = FIL (PVB − UAL − AAV)

= EAN PVNC

$$\text{FIL NC} = \frac{\text{FIL PVNC}}{\text{avg } \ddot{a}_{x:\overline{RA-x|}}} = \frac{\text{EAN PVNC}}{\text{average annuity}}$$

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2 Entry age normal is an individual cost method. This problem is mainly testing handling of commutation functions with salary scales.

$EANC\% = PVB_{EA} / PVE_{EA} = PVB_{EA} / [(\text{Pay at } EA) * ({}^{s}\ddot{a}_{EA:\overline{RA-EA}|})]$

$EANC_{CA} = EANC\% * \text{Pay at } CA$

$= (1+s)^{CA-EA} [PVB_{EA} / {}^{s}\ddot{a}_{EA:\overline{RA-EA}|}]$

$PVB_{EA} = (\text{Projected Benefit})\ \ddot{a}^{(12)}_{RA}\ D_{RA}/D_{EA}$

$EAN\ AL = (EANC_{CA})({}^{s}\ddot{s}_{EA:\overline{CA-EA}|})$ retrospective definition

1-1-94 Age 34   Age 34 pay $= 25{,}000$ Age 64 pay $= 25{,}000(1.05)^{30}$

Entry age 30   Age 64 FAE $=$ Age 64 pay $(\ddot{a}_{\overline{5}|1.05}/5)$

50% Projected benefit $= .5(25{,}000)(1.05)^{30}(\ddot{a}_{\overline{5}|.05}/5)$

$= 49{,}118$

PVB at entry age 30 (50%) $= 49{,}118(8.776)(94/1262) = 32{,}108$

EANC at age 34 (50%) $= 32{,}108(1.05)^4 / {}^{s}\ddot{a}_{30:\overline{35}|}$

$= 32{,}108(1.2155)({}^{s}N_{30} - {}^{s}N_{65}) / {}^{s}D_{30}$

$= 32{,}108(1.2155)(164{,}704 - 30{,}013)/5{,}454$

$= 1{,}580$

EAN AL at age 34 (50%) $= 1{,}580({}^{s}N_{30} - {}^{s}N_{34}) / {}^{s}D_{34}$ retrospective

$= 1{,}580(164{,}704 - 143{,}532)/5{,}033$

$= 6{,}648$

EAN AL at age 34 (60%) $= 6{,}648(60/50) = 7{,}978$

Δ EAN AL $= 7{,}978 - 6{,}648 = 1{,}330$

A

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3 The Aggregate cost method is clearly an aggregate type cost method. This problem requires you to do cost calculations with both a 5% and a 6% salary scale. The first step is calculation of projected benefits with both salary scales:

1-1-94 Age 60 Past service 25 Age 60 pay = 100,000
Hire age 35 Total service 30

	5% salary scale	6% salary scale
Age 64 pay	$100{,}000(1.05)^4$ $= 121{,}551$	$100{,}000(1.06)^4$ $= 126{,}248$
$FAE\overline{5\rvert}$ factor	$\ddot{a}_{\overline{5\rvert}5\%}/5 = .9092$	$\ddot{a}_{\overline{5\rvert}6\%}/5 = .8930$
$FAE\overline{5\rvert}$ at 65	110,513	112,742
Projected Benefit	55,256	56,371

Now you can solve for the PV of future benefits at 1-1-94 based on the 5% salary scale:

PVNC = PVB − AAV
PVB = PVNC + 200,000

NC = 38,120 = PVNC/(average PVE/E)
$PVNC = 38{,}120(\ddot{a}_{\overline{5\rvert}j})$
where $1+j = (1.07/1.05) = 1.0190$
PVNC = 38,120(4.8165) = 183,607
PVB = 383,607

The PVB under the 6% salary scale assumption will reflect the pro-rata difference in the projected benefits:

PVB (6% sal) = (56,371/55,256)(383,607) = 391,345
PVNC = 391,345 − 200,000 = 191,345
NC = $191{,}345/\ddot{a}_{\overline{5\rvert}k}$ where $1+k = 1.07/1.06 = 1.0094$
= 191,345/4.9074
= 38,991

B

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4 Under the Projected Unit Credit method, the accrued liability is defined as the present value of a "funding" accrued benefit. This benefit is calculating by applying the rates of benefit accrual to years of past service, and using projected earnings to ARA.

One minor trick to the problem is calculating $\ddot{a}_{65}^{(12)}$:

$$\ddot{a}_{65}^{(12)} = \ddot{a}_{65} - 11/24 = 868{,}052/94{,}414 - 11/24 = 8.7358$$

$$\text{PUC AL} = \text{FAB}\ \ddot{a}_{65}^{(12)}\ D_{65}/D_x$$

	Smith	Brown
1-1-94 Age x	50	50
Hire age	25	40
Past service	25	10
Pay at current age	50,000	50,000
Pay at age 64	$(1.05)^{14}$ 50,000 = 98,997	$(1.05)^{14}$ 50,000 = 98,997
FAE3 at age 64	$(\ddot{a}_{\overline{3}|5\%}/3)$ 98,997 = 94,357	$(\ddot{a}_{\overline{3}|5\%}/3)$ 98,997 = 94,357
FAB accrual	.02(20) + .01(5)	.02(10)
FAB	(.45) 94,357	(.20) 94,357
	= 42,461	= 18,871

$$\text{PV factor at age 50} = \ddot{a}_{65}^{(12)}\ D_{65}/D_{50}$$
$$= 8.7358(94{,}415/310{,}647)$$
$$= 2.6550$$

PUC AL	2.6550(42,461)	2.6550(18,871)
	= 112,735	= 50,104
		Σ = 162,839

D

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5 Under Projected Unit Credit, the accrued liability is defined as the present value of a "funding" accrued benefit. This benefit is calculated by applying the rates of benefit accrual to years of past service, and using projected earnings to ARA.

With a uniform accrual rate for all years, the FAB is easily calculated. By changing from assumed retirement at age 65 to half at age 62 and half at age 65, the problem requires you to calculate liabilities at ages 62 and 65.

1-1-94 Age 50 Past service 20
Hire age 30 Age 50 pay 50,000

	Retirement Age 62	Retirement Age 65
Projected pay at RA	$50{,}000(1.05)^{12} = 85{,}517$	$50{,}000(1.05)^{14} = 98{,}997$
FAE3 at RA	$85{,}517(\ddot{a}_{\overline{3}|5\%}/3) = 81{,}509$	$98{,}997(\ddot{a}_{\overline{3}|5\%}/3) = 94{,}357$
Early reduction factor	$1 - 3(.03) = .91$	1.00
FAB including ERF	$20(2\%)(.91)(81{,}509)$ $= 29{,}669$	$20(2\%)(1.00)(94{,}357)$ $= 37{,}743$

With retirement decrements, UC $AL = \sum_{t=0}^{65-x} v^t \, {}_tp_x \, q_{x+t}^{(r)} \, \ddot{a}_{x+t}^{(12)}$ Reduced FAB_{x+t}

(1) t	(2) Age $x+t$	(3) $q_{x+t}^{(r)}$	(4) ${}_tp_x$	(5) Reduced FAB_{x+t}	(6) $\ddot{a}_{x+t}^{(12)}$	(7) v^t	Product (3)(4)(5)(6)(7)
12	62	.50	1.00	29,669	9.394	$(1.07)^{-12}$	61,876
13	63	—					
14	64	—					
15	65	1.00	.50	37,743	8.736	$(1.07)^{-15}$	59,753
						Σ	121,629

For ARA = 65 only, PUC $AL = 2(59{,}753) =$ 119,507

$\Delta = 2{,}123$ A

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6 Under the Projected unit credit method, the accrued liability is defined as the present value of a "funding" accrued benefit. This benefit is calculated by applying the rates of benefit accrual to years of past service, and using <u>projected</u> earnings to ARA.

The calculation of retirement age G/L is based on the difference between the accrued liability as an active employee and as a retiree.

1-1-94 Age	60	Age 59 pay	40,000
Hire age	40	Age 58 pay	38,095
Past service	20	Age 57 pay	<u>36,281</u>
Spouse age	60	FAE 3 at 60	114,376 ÷ 3 = 38,125

<u>PVB = AL as retiree</u>

Age 60 early ret benefit $= .02(20)(38{,}125) = 15{,}250$

PV factor-optional form $= \ddot{a}^{(12)}_{60} + 1.0\left(\ddot{a}^{(12)}_{60} - \ddot{a}^{(12)}_{60:60}\right)$

$= 9.815 + 9.815 - 8.094 = 11.5360$

PVB as retiree $= .95(15{,}250)\,11.5360$

$= 167{,}129$

<u>AL as active employee</u>

Age 64 pay $= 40{,}000(1.05)^5 = 51{,}051$

FAE 3 at age 65 $= 51{,}051(\ddot{a}_{\overline{3}|.05}/3) = 48{,}659$

Funding Accrued Benefit $= 48{,}659(.02)(20)$

$= 19{,}464$

PUC AL $= 19{,}464\ \ddot{a}^{(12)}_{65}\ D_{65}/D_{60}$

$= 19{,}464\,(8.736)(1.07)^{-5}$ no pre-ret decrements

$= 121{,}231$

$\Delta AL = 45{,}898$

C

Spring 1994 EA-1B Solutions Solution Added 2/12/2000

1 When you have both a salary scale and withdrawal decrements, the annuities are difficult to calculate. You could calculate the EAN AL on either a prospective or retrospective basis. The preferred approach is to use retrospective, and avoid calculating annuity values entirely. It is clearer to accumulate the prior year's accrued liability one year at a time.

Since Smith was hired at 1-1-94, you can ignore them in this problem - the AL for Smith is zero at 1-1-94.

Brown 1-1-94 Age 38
Hire age 35

$$EAN\ AL = EANC_{CA}\left(\frac{{}^{s}N_{EA} - {}^{s}N_{CA}}{{}^{s}D_{CA}}\right)$$

$$AL_1 = \frac{{}^{s}D_x}{{}^{s}D_{x+1}}\left(EANC_{CA} + AL_x\right)$$

$$= \frac{(1+i)}{p_x^{(T)}(1+s)}\left(EANC_{CA} + AL_x\right)$$

Date	Age x	$p_x^{(T)}$	$EANC_{38}$	AL_x	$AL_{x+1} = (1.07)(16{,}000 + AL_x)/[(p_x^{(T)})(1.04)]$
1-91	35	.75	16,000	0	21,949 = 1.07(16,000 + 0)/[.75(1.04)]
1-92	36	.80	16,000	21,949	48,804 = 1.07((16,000+21,949)/[.80(1.04)]
1-93	37	.85	16,000	48,804	78,440 = 1.07(16,000+48,804)/[.85(1.04)]
1-94	38	.90	16,000	78,440	

(B)

The alternative solution is to determine values for ${}^{s}D_x^{(T)}$ and ${}^{s}N_x^{(T)}$ to do a direct calculation of the accrued liability as shown above:

$$EAN\ AL = EANC_{CA}\left(\frac{{}^{s}N_{EA} - {}^{s}N_{CA}}{{}^{s}D_{CA}}\right)$$

See the next page for the details of the alternate solution

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(7) Entry Age Normal is an individual cost method. The formula definitions for EAN are

$$EANC\% = PVB_{EA} / PVE_{EA}$$

$$EANC_{CA} = EANC\% \text{ (Pay at CA)}$$

$$EAN\ AL = EANC_{CA}\left({}^{s}\ddot{s}_{EA:\overline{CA-EA}|}\right) \quad \text{retrospective definition}$$
$$= EANC_{CA}\left(\frac{{}^{s}N_{EA} - {}^{s}N_{CA}}{{}^{s}D_{CA}}\right)$$

In this problem you are given the EANC values for Smith and Brown as of 1-1-94. To calculate the EAN accrued liability, you must derive the ${}^{s}D_x^{(T)}$ factors including the withdrawal decrements. Since Smith was hired at 1-1-94, you do not need to do any calculations (zero accrued liability).

Brown 1-1-94 age 38 $\qquad {}^{s}D_x^{(T)} = v^x \, l_x^{(T)} (1+s)^x$

Hire age 35 $\qquad s = 4\%$ salary scale

Age x	$q_x^{(w)}$	$p_x^{(T)}$	${}_{x-35}p_{35}^{(T)}$	$l_x^{(T)}$	v^x	$(1+s)^x$	${}^{s}D_x^{(T)}$
35	.25	.75	1.00	100.0	.0937	3.9461	.3696
36	.20	.80	.75	75.0	.0875	4.1039	.2694
37	.15	.85	.60	60.0	.0818	4.2681	.2095
38	.10	.90	.51	51.0	.0765	4.4388	.1731

$$\text{Brown EAN AL} = 16{,}000\left({}^{s}N_{35}^{(T)} - {}^{s}N_{38}^{(T)}\right) / {}^{s}D_{38}^{(T)}$$
$$= 16{,}000\left({}^{s}D_{35}^{(T)} + {}^{s}D_{36}^{(T)} + {}^{s}D_{37}^{(T)}\right) / {}^{s}D_{38}^{(T)}$$
$$= 16{,}000\,(.3696 + .2694 + .2095) / .1731$$
$$= 78{,}440$$

(B)

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8 Entry Age Normal is an individual cost method. The Entry Age normal cost is defined as PVB_{EA}/PVL_{EA} (when the benefit is not based on pay). In this problem, both the interest rate and the plan benefits change at 1-1-94. You are told to calculate the change in EAN AL due to the decrease in interest rate before the change in EAN AL due to the plan amendment.

1-1-94 Age 41 Past service 14
Entry age 27 Total service 38

$$EANC = PVB_{EA} / PVL_{EA}$$
$$= \frac{(\text{Projected benefit})\, \ddot{a}_{65}^{(12)}\, D_{65}/D_{27}}{\ddot{a}_{27:\overline{38|}}}$$
$$= (\text{Projected benefit})\, \ddot{a}_{65}^{(12)}\, v^{38} / \ddot{a}_{\overline{38|}i} \quad \text{(no pre-ret decrements)}$$
$$EAN\ AL = EANC(\ddot{s}_{27:\overline{14|}}) \quad \text{retrospective definition}$$
$$= (\text{Projected benefit})\, \ddot{a}_{65}^{(12)}\, (\ddot{s}_{\overline{14|}i} / \ddot{s}_{\overline{38|}i})$$

\$15 Projected benefit (monthly) = 38(\$15) = \$570

Accrued liability with \$15:	6% i	7% i	
$12(\ddot{a}_{65}^{(12)})$	112.14	104.83	
$12(\ddot{a}_{65}^{(12)})570$ = PVB at 65	63,920	59,753	
$\ddot{s}_{\overline{14|}i}$	22.2760	24.1290	
$\ddot{s}_{\overline{38|}i}$	144.0585	184.6403	
EAN AL	9,884	7,809	Δ = 2,075

Δ EAN AL due to \$18 benefit, using 6% interest is
(9,884)(18/15) − 9,884 = 9,884(3/15) = 1,977

Difference between two deltas = 2,075 − 1,977 = 98

A

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9 Entry Age Normal is an individual cost method. For a plan where benefits are not based on pay, the normal cost is defined as $(PVB_{EA} / \ddot{a}_{EA:\overline{RA-EA}|})$.

The trick to the problem is correctly handling the withdrawal decrement at age 50, plus knowing when to use the commutation functions that you are given. The commutation functions should only be used to calculate present values of post-withdrawal liabilities. As long as a participant is active, there are no mortality decrements assumed.

1-1-94 Age 40   Past service 10
Entry age 30   Total service 35

PVB_{EA} = (Projected Benefit) $\ddot{a}_{65}^{(12)}\ D_{65}/D_{30}$ (pre-term assumptions)
$+ \sum_{t=0}^{34} v^t\ {}_tp_{30}\ q_{30+t}^{(w)}$ $(\text{Benefit})_{30+t}(\ddot{a}_{65}^{(12)}\ D_{65}/D_{30+t})$ (post-term)

${}_tp_{30} = 1$ when $t \le 19$, ${}_tp_{30} = .70$ when $t \ge 20$

$$PVB_{EA} = \$50(12)(35)\ \ddot{a}_{65}^{(12)}\ (.70)\ v^{35} \quad \text{retirement}$$
$$+\ v^{20}(1.0)(.30)[(\$50)(12)(20)]\ \ddot{a}_{65}^{(12)}\ D_{65}/D_{50} \quad \text{withdrawal}$$
$$= 21{,}000\,(8.7)\,(.7)\,(1.07)^{-35}$$
$$+\ (1.07)^{-20}\,(.3)\,(12{,}000)(8.7)(94/311)$$
$$= 11{,}979 \text{ retirement} + 2{,}446 \text{ withdrawal} = 14{,}425$$

$$\ddot{a}_{30:\overline{35}|} = 1 + v\,p_{30} + v^2\,{}_2p_{30} + \ldots + v^{34}\,{}_{34}p_{30}$$
$$= 1 + v + v^2 + \ldots + v^{19} + .70(v^{20} + v^{21} + \ldots + v^{34})$$
$$= .7(1 + v + \ldots + v^{19} + \ldots + v^{34}) + .3(1 + v + v^2 + \ldots + v^{19})$$
$$= .7\ \ddot{a}_{\overline{35}|.07} + .3\ \ddot{a}_{\overline{20}|.07} = 13.0985$$

EANC = 14,425 / 13.0985 = 1,101   (E)

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10 Unit Credit is an individual cost method. The accrued liability is defined as the present value of the accrued benefit.

With retirement decrements, the present value must be calculated as a summation (the standard approach used for ancillary benefits). The trick to this problem is that some retirements are assumed to occur after normal retirement age. You must apply the actuarial increase factor $(N_{65}^{12}/N_x^{(12)})$ to benefits which are valued after normal retirement age - this is the definition of the postponed retirement benefit in this problem.

1-1-94 Age 65   Past service 24
Entry age 41   Normal retirement ben = \$20(12)(24) = \$5,760

UC AL = PV of accrued benefit

$$= \sum_{t=0}^{2} v^t \, {}_tp_x \, q_{x+t}^{(r)} \, \ddot{a}_{x+t}^{(12)} \, (\text{Ret Ben})_{x+t}$$

t	Age $x+t$	(1) $q_{x+t}^{(r)}$	(2) ${}_tp_{65}^{(T)}$	(3) Ben_{x+t}	(4) $\ddot{a}_{x+t}^{(12)}$	(5) v^t	Product (1)(2)(3)(4)(5)
0	65	.60	1.00	5760	$\frac{825}{94}$	$(1.07)^0$	.6(1.0)(5760)(825/94)(1.0) = 30,332
1	66	.80	.40	$5760\left(\frac{825}{734}\right)$	$\frac{734}{86}$	$(1.07)^{-1}$	.8(.40)(5760)(825/86)(1.07)$^{-1}$ = 16,525
2	67	1.00	.08	$5760\left(\frac{825}{651}\right)$	$\frac{651}{79}$	$(1.07)^{-2}$	1.0(.08)(5760)(825/79)(1.07)$^{-2}$ = 4,203

$\Sigma = 51,060$

C

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11 You can use the non-investment G/L formula to calculate the amount of the mortality gain. You will need to solve for the interest rate based on the relationship between annuities at successive ages:

$$v\,p_x\,\ddot{a}_{x+1} = a_x = \ddot{a}_x - 1.0$$

$$p_x\,\ddot{a}_{x+1} = (1+i)(\ddot{a}_x - 1.0)$$

$$1+i = \frac{p_x\,\ddot{a}_{x+1}}{\ddot{a}_x - 1.0}$$

$$= \frac{p_{70}\,\ddot{a}_{71}}{\ddot{a}_{70} - 1.0} = \frac{.9636(8.80)}{9.0 - 1.0} = 1.0600$$

non-investment G/L = $eAL_1 - AL_1$

1-1-93 Age 70
Spouse age 65

$$eAL_1 = (1+i)(AL_0 + NC_0) - (\text{actual } BP+i)$$

$$= 1.06(AL_0 + \emptyset) - 1.06(20{,}000) \quad \text{zero NC for retiree}$$

$$= 1.06(20{,}000)\left[\ddot{a}_{70} + \tfrac{1}{2}(\ddot{a}_{65} - \ddot{a}_{65:70}) - 1\right]$$

$$= 1.06(20{,}000)(9 + .5(10-8) - 1)$$

$$= 190{,}800$$

1-1-94 Age 71
Spouse died

$$AL_1 = 20{,}000\,\ddot{a}_{71}$$

$$= 20{,}000(8.8)$$

$$= 176{,}000$$

$G/L = {}_eAL_1 - AL_1 = 190{,}800 - 176{,}000 = 14{,}800$ B

Spring 1994 EA-1B Solutions

12 Entry Age Normal is an individual cost method. The Entry Age normal cost as a percentage of pay is PVB_{EA} / PVE_{EA}. This problem tests your ability to calculate values using commutation functions with salary scales.

$$EANC\% = PVB_{EA}/PVE_{EA} = PVB_{EA}/[(\text{Pay at }EA)\,{}^{s}\ddot{a}_{EA:\overline{RA-EA}|}]$$

$$EANC_{CA} = EANC\% * \text{Pay at } CA$$

$$= (1+s)^{CA-EA}\left[PVB_{EA} / {}^{s}\ddot{a}_{EA:\overline{RA-EA}|}\right]$$

$$EAN\ AL = (EANC_{CA})\,{}^{s}\ddot{s}_{EA:\overline{CA-EA}|} \quad \text{retrospective definition}$$

$$= (1+s)^{CA-EA}(PVB_{EA})\frac{{}^{s}\ddot{s}_{EA:\overline{CA-EA}|}}{{}^{s}\ddot{a}_{EA:\overline{RA-EA}|}}$$

1-1-94 Age 50 — Age 50 pay = 53,000

Entry age 40 — Age 64 pay = $53{,}000(1.06)^{14}$ = 119,828

FAE5 at 65 = $119{,}828\,(\ddot{a}_{\overline{5}|.06}/5)$ = 107,009

Projected benefit = 107,009 (.50) = 53,504

$$EAN\ AL = (1.06)^{10}(53{,}504)\,\ddot{a}^{(12)}_{65}\frac{D_{65}}{D_{40}}\left[\frac{({}^{s}N_{40}-{}^{s}N_{50})/{}^{s}D_{50}}{({}^{s}N_{40}-{}^{s}N_{65})/{}^{s}D_{40}}\right]$$

$$= (1.06)^{10}(53{,}504)(8.74)\left(\frac{7{,}448}{49{,}876}\right)\left[\frac{(15{,}607{,}843-10{,}748{,}428)/451{,}387}{(15{,}607{,}843-4{,}770{,}425)/513{,}015}\right]$$

$$= 837{,}445(.1493)\left[\frac{10.7655}{21.1250}\right]$$

$$= 63{,}730$$

C

Spring 1994 EA-1B Solutions

13 Attained Age Normal is an aggregate cost method. In the first year, the UAL is calculated based on the Unit Credit cost method. After the first year, the UAL is defined equal to eUAL.

You must reproduce the valuation results at 1-1-93 to determine the normal cost. Then apply the formula to calculate eUAL at 1-1-94. One minor trick in the problem is computation of $\ddot{a}_{65}^{(12)}$:

$$\ddot{a}_{65}^{(12)} = \ddot{a}_{65} - 11/24 = N_{65}/D_{65} - 11/24$$
$$= 263{,}044/28{,}610 - 11/24$$
$$= 8.7358$$

1-1-93 Age 49
Past Service 1
Total Service 17

Accrued benefit $= 1(\$20)(12) = \$240$

U.C. AL $=$ PV of accrued benefit
$$= 240\,\ddot{a}_{65}^{(12)}\, D_{65}/D_{49}$$
$$= 240(8.7358)(28{,}610/101{,}24$$
$$= 592.48$$

<u>AAN calculations</u>

NC = PVNC / average PVL

PVNC = PVB − AAV − UAL = PVB − 0 − 592

$$PVB = (\text{Projected benefit})\,\ddot{a}_{65}^{(12)}\, D_{65}/D_{49}$$
$$= [17(20)(12)]\,\ddot{a}_{65}^{(12)}\, D_{65}/D_{49}$$
$$= (17/1)\,592.48$$
$$= 10{,}072$$

$$PVNC = 10{,}072 - 592 = 9480$$

$$NC = 9{,}480 / \ddot{a}_{49:\overline{16}|}$$
$$= 9{,}480 / [(N_{49} - N_{65})/D_{49}] =$$
$$= 9{,}480/[(1{,}238{,}268 - 263{,}044)/101{,}241] = 9480/9.6327 = 984$$

1-1-94 UAL = $_e$UAL $= (1+i)(NC_0 + UAL_0) - (C+I) = 1.07(984+592) - 1125 = 562$ B

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14 Frozen Initial Liability is an aggregate cost method. It is unusual to see a G/L question, since you normally don't calculate the amount of G/L under aggregate type cost methods.

In general, if all actuarial assumptions are matched by experience under the plan, the normal cost will be level as a dollar amount (or percent of payroll, if benefits are based on pay). With no decrements prior to retirement, the only source of G/L in this plan is investment experience. You can calculate the normal cost each year, and determine the G/L indirectly.

1-1-93 Age 51
Future service 14

* Sum of UAL plus AAV

$PVNC = 122{,}000 - 65{,}000$ *

$= 57{,}000$

$\ddot{a}_{51:\overline{14}|} = \ddot{a}_{\overline{14}|.07}$ no pre-ret decrements

$= 9.3577$

$NC = 57{,}000 / 9.3577$

$= 6{,}091.27$

1-1-94 Age 52

$PVB = 1.07(122{,}000)$ no pre-ret decrements

$PVNC = 1.07(122{,}000) - 76{,}000$

$= 54{,}540$

$\ddot{a}_{52:\overline{13}|} = \ddot{a}_{\overline{13}|.07}$

$= 8.9427$

If all assumptions were met, the normal cost would still be 6091.27, and the expected PVNC is

$ePVNC = 6091.27(8.9427) = 54{,}472$

PVNC actual $54{,}540$

Δ 68 loss

B

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(14) The harder way to work the problem is to directly calculate the expected value of assets, and the investment G/L:

$eAAV_1 = (1+i)(AAV_0) + (\text{contrib} + I) - (\text{benefit pmt} + I)$

$eUAL_1 = (1+i)(NC_0 + UAL_0) - (\text{contrib} + I) = UAL_1$ by definition

$48{,}500 = 1.07(6091 + UAL_0) - (\text{contrib} + I)$

$$UAL_0 = \frac{48{,}500 + \text{contrib} + I}{1.07} - 6091$$

Assume contribution is paid at the beginning of the year, so $(\text{contrib} + I) = 1.07(C)$

$$UAL_0 = \frac{48{,}500 + 1.07(C)}{1.07} - 6091 = 39{,}236 + C$$

$\therefore \quad AAV_0 = 65{,}000 - (39{,}236 + C) = 25{,}764 - C$

$eAAV_1 = 1.07(25{,}764 - C) + 1.07(C)$ again, contrib assumed BOY

$= 27{,}568$

$AAV_1 = 27{,}500$

$\Delta =$ 68 loss

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15 Individual Aggregate is a hybrid of the Aggregate method and the Individual Level Premium method. The normal cost is calculated on an individual basis, and there is no UAL. Normally you would allocate assets based on a formula given in the problem. Since this plan has just been established, there are no assets to allocate.

The main point of this problem is the handling of the retirement decrements. You have to do this both for the calculation of the present value of benefits, and for the temporary annuity to calculate the normal cost.

1-1-94 Age 45 $\quad PVB = \sum_{t=18}^{20} v^t \, {}_tp_x \, q_{x+t}^{(r)} \, \ddot{a}_{x+t}^{(12)} \, (\text{RetBen})_{x+t}$

t	Age $x+t$	(1) $q_{x+t}^{(r)}$	(2) ${}_tp_x^{(T)}$	(3) v^t	(4) $\ddot{a}_{x+t}^{(12)}$	(5) $(\text{RetBen})_{x+t}$	Product (1)(2)(3)(4)(5)
18	63	.3333	1.0000	$(1.07)^{-18}$	8.96	12(900-100)	8,482
19	64	.6667	.6667	$(1.07)^{-19}$	8.74	12(900-50)	10,957
20	65	1.0000	.2222	$(1.07)^{-20}$	8.51	12(900-0)	5,278
							24,717 PVB @ 45

With no decrements, $\ddot{a}_{45:\overline{20|}} = \ddot{a}_{\overline{20|}} = 1 + v + \dots + v^{18} + v^{19}$

The trick to this problem is that the retirement decrements occur at the beginning of the year. For purpose of payments towards the normal cost, there would be .6667 people left at age 63, and .2222 people left at age 64:

$$\ddot{a}_{45:\overline{20|}} = 1 + v + \dots + v^{17} + .6667\,v^{18} + .2222\,v^{19}$$
$$= \ddot{a}_{\overline{20|}.07} - .3333\,(1.07)^{-18} - .7778\,(1.07)^{-19}$$
$$= 11.3356 - .0986 - .2151 = 11.0219$$

$$NC = \frac{PVNC}{\ddot{a}_{45:\overline{20|}}} = \frac{PVB - 0}{\ddot{a}_{45:\overline{20|}}} = \frac{24{,}717}{11.0219} = 2242.$$

B

Revised 2/12/2000

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16 Individual Level Premium is an individual cost method. The normal cost is defined as the sum of multiple layers which are established each time that the projected benefit changes.

In this problem, you must calculate the projected benefit (and the change) each time the employee's earnings change. The real trick to this problem is correctly calculating the FAE3 when the employee gets closer to normal retirement age.

1-1-94 Age 64 FAE3 at 65 = $\frac{1}{3}$ (Age 62 pay + Age 63 pay + Age 64 pay)

Valuation date	1-1-90	1-1-92	1-1-94
Age	60	62	64
Pay	60,000	80,000	65,000
Projected pay at 64	60,000	80,000	65,000
Proj high 3 yr comp @ 65	60,000	80,000	75,000 *
Projected benefit	30,000	40,000	37,500
Δ Projected benefit	30,000	10,000	−2,500
Δ PVB at valn date	$30{,}000\,v^5\,\ddot{a}_{65}^{(12)}$	$10{,}000\,v^3\,\ddot{a}_{65}^{(12)}$	$-2{,}500\,v\,\ddot{a}_{65}^{(12)}$
Δ NC at valn date	$\dfrac{30{,}000\,\ddot{a}_{65}^{(12)}}{\ddot{s}_{\overline{5}\rvert .07}}$	$\dfrac{10{,}000\,\ddot{a}_{65}^{(12)}}{\ddot{s}_{\overline{3}\rvert .07}}$	$\dfrac{-2{,}500\,\ddot{a}_{65}^{(12)}}{\ddot{s}_{\overline{1}\rvert .07}}$
	$= \dfrac{30{,}000(8.786)}{6.1533}$	$= \dfrac{10{,}000(8.786)}{3.4399}$	$= \dfrac{-2{,}500(8.786)}{1.07}$
	= 42,836	= 25,541	= −20,528

1-1-94 Normal cost = 42,836 + 25,541 − 20,528
= 47,849

(C)

* Calculated as $\frac{1}{3}$ (80,000 + 80,000 + 65,000)
= 75,000

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17 Under side fund problems, you must calculate the net benefit which is not funded by the insurance policies. Then you apply the underlying actuarial cost method to determine the normal cost required to fund the net benefit. This normal cost is called the side fund normal cost. If you add the insurance premiums to the side fund normal cost, the result is called the total normal cost.

Total projected benefit	24,000	1-1-94 Age 45
CSV at ARA	50,000	
Equivalent annual benefit	5,695 = 50,000 / 8.78	
Net benefit not funded through life insurance	18,305 = 24,000 - 5,695	
Aggregate PVB	$18{,}305\ \ddot{a}_{65}^{(12)}\ D_{65}/D_{45}$ $= 18{,}305(8.78)(94/445)$ $= 33{,}950$	
Aggregate PVNC	23,950 = 33,950 - 10,000 AAV	
$\ddot{a}_{x:\overline{65-x}\rvert}$	$= (N_{45} - N_{65}) / D_{45}$ $= (5691 - 868) / 445$ $= 10.8382$	
Side fund normal cost	$= 23{,}950 / 10.8382$ $= 2{,}210$	
Insurance premiums	2,000	
Total normal cost	4,210 = 2,210 + 2,000	

B

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18 Frozen Initial Liability is an aggregate cost method. The expected unfunded liability formula is used to calculate each years UAL after the first year. The first year's UAL is determined under the Entry Age Normal method.

This is an aggregate G/L question, which involves projections from one valuation date to the next. Unlike most other problems of this type, this is an extremely easy problem. In this problem, you are told that there are no experience gains or losses from any source. That also means that the normal cost should be the same percent of payroll at 1-1-94 and 1-1-93.

1-1-93 PVNC = PVB - UAL - AAV
= 500,000 - 100,000 - 100,000
= 300,000

NC% = PVNC / PVE
= 300,000 / 3,000,000
= 10.0%

1-1-94 Expected pay = 1.05(200,000)
= 210,000

Expected NC = 210,000 (10.0%)
= 21,000

C

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19 The calculation of the present value of benefits is complicated by the retirement decrements. Once you write down the expression for the PVB, you can set up multiple columns of data to do the calculation.

1-1-94 Age 50 $\quad PVB = \sum_{10}^{15} v^t \, {}_tp_x \, q_{x+t}^{(r)} \, \ddot{a}_{x+t}^{(12)} \, (\text{Ret Ben})_{x+t}$

Service 10

t	Age $x+t$	Service	(1) $q_{x+t}^{(r)}$	$p_{x+t}^{(T)}$	(2) ${}_tp_x^{(T)}$	(3) v^t	(4) $(\text{RetBen})_{x+t}$	(5) $\ddot{a}_{x+t}^{(12)}$	(1)(2)(3)(4)(5) Product
10	60	20	.20	.80	1.00	$(1.07)^{-10}$	20(120)(1-5(.03))	9.815	2,036
11	61	21	.20	.80	.80	$(1.07)^{-11}$	21(120)(1-4(.03))	9.607	1,619
12	62	22	.50	.50	.64	$(1.07)^{-12}$	22(120)(1-3(.03))	9.394	3,207
13	63	23	—	1.00	.32	—	—	—	—
14	64	24	—	1.00	.32	—	—	—	—
15	65	25	1.00	—	.32	$(1.07)^{-15}$	25(120)(1)	8.736	3,040
									9,901

(B)

Since there are no retirement decrements at ages 63 and 64, you don't need to waste time doing any calculations at those ages.

The tricky part of this problem is the correct calculation of ${}_tp_x^{(T)}$, which is the product of last year's ${}_tp_x^{(T)}$ and last year's $p_{x+t}^{(T)}$.

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20 Unit Credit is an individual cost method. The accrued liability is defined equal to the present value of the accrued benefit.

This problem appears to be similar to ones in the past that can be worked quickly using a shortcut. The trick is that you can't use the shortcut (eliminating the withdrawal decrement because of 100% vesting under Unit Credit), and the reason is that you have NOT got the same assumptions before and after termination. There is no mortality assumed for actives, but there are mortality decrements in the commutation functions used to value liabilities for terminations.

You must set up an expression for the AL which values retirement benefits and termination benefits:

UCAL = Retirement $2400(.90)(8.7)(1.07)^{-25}$

$= AB(\ddot{a}_{65}^{(12)})(D_{65}^{(T)}/D_x^{(T)})$

+ Termination $2400(.10)(1.07)^{-10}(8.7)(94/311)$

$= AB\, v^{10}\, {}_{10}p_{40}\, q_{50}^{(t)}(\ddot{a}_{65}^{(12)})(D_{65}/D_{50})$

$= 2400(.9)(8.7)(.1842) + 2400(.1)(.5083)(8.7)(94/311)$

$= 3462 + 321$

$= 3783$

B

If you try the shortcut, you get the wrong answer

$2400\, \ddot{a}_{65}^{(12)} v^{25} = 3847$