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SPRING 1993 EA-1B EXAM SOLUTIONS

Revision History:
02/12/00 Corrected problem 19 – clarified solution

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1 Individual entry age normal is an individual cost method. This problem is a workout in use of salary scales, with plenty of arithmetic (NOT! zero sal scale!)

By definition, the accrued liability equals the present value of future benefits less the present value of future normal costs. You can check your work by using the retrospective definition, which equals the accumulated value of past normal costs.

$$AL = PVFB - PVNC$$
$$= (\text{Proj Ben})\, \ddot{a}_{65}^{(12)} \frac{D_{65}}{D_x} - EANC\% \,(\text{PV Earnings})$$

$$EANC\% = \frac{PVB_{EA}}{PV\ Earnings_{EA}}$$

	Smith	Brown
Date of birth	1-1-56	1-1-36
1-1-93 Age	37	57
Date of hire	1-1-92	1-1-76
Entry Age	36	40
Service at 65	29	25
1993 compensation	30,000	18,000
Projected age 64 comp	$30{,}000(1.0)^{27}$	18,000
Projected benefit	15,000	9,000
PVB @ entry age	$150{,}000(1.07)^{-29}$ = 21,084	$90{,}000(1.07)^{-25}$ = 16,582

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(1) With no salary scale, you can simply calculate the EANC as a level \$ amount:

$$EANC = \frac{PVB_{EA}}{\ddot{a}_{\overline{EA:RA-EA|}}}$$

	Smith	Brown	
$\ddot{a}_{\overline{EA:65-EA|}}$	$\ddot{a}_{\overline{29|}.07}$ = 13.1371	$\ddot{a}_{\overline{25|}.07}$ = 12.4693	
EANC	21,084/13.1371 = 1,605	16,582/12.4693 = 1,330	
PVB @ current age	$150{,}000(1.07)^{-28}$ = 22,560	$90{,}000(1.07)^{-8}$ = 52,381	
PV(EANC)	$1605\,\ddot{a}_{\overline{28|}.07}$ = 20,843	$1330\,\ddot{a}_{\overline{8|}.07}$ = 8,497	
AL = PVB − PVNC	1,717	43,884	Σ = 45,601

(C)

You can now use the retrospective definition as a check. It will not protect you against any arithmetic errors in the definition of the entry age normal cost:

	Smith	Brown
$\ddot{s}_{\overline{EA:CA-EA|}}$	$\ddot{s}_{\overline{1|}.07}$ = 1.07	$\ddot{s}_{\overline{17|}.07}$ 32.9990
accum EANC	1.07(1605) = 1,717 ✓	32.999(1330) = 43,884 ✓

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2 The key to working this problem is to adjust the UAL for three changes:
(i) change in normal retirement benefit
(ii) change in asset valuation method
(iii) change in assumed interest rate

You are given the change in the entry age normal accrued liability due to the change in interest rate. You can adjust the 8% accrued liability to reflect the change in normal retirement benefit:

7% AL on \$15 benefit : 270,000
8% AL on \$15 benefit : 225,000
8% AL on \$20 benefit : 300,000 $= \left(\frac{20}{15}\right) 225{,}000$

The change in the UAL due to the change in asset valuation methods is -25,000. The net change in the UAL for all three changes is 300,000 - 270,000 - 25,000 = 5,000.

Under the FIL method, the normal cost is calculated on an aggregate basis. You'll have to adjust the PVB to reflect the \$20 benefit:

PVB = 350,000 (20/15) = 466,667
UAL = 60,000 + 5,000 = 65,000
AAV = 250,000
PVNC = PVB - UAL - AAV = 151,667
Average annuity = 10
NC = 15,167

E

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3 You are given the values of certain mortality gains. These can be determined using the formula for non-investment gains and losses:

$$\text{non-inv } G/L = eAL_1 - AL_1$$

AL_1 is defined based on who is alive

$$eAL_1 = (1+i)(NC_0 + AL_0) - (\text{actual } BP + i)$$

For non-actives, the NC_0 term is zero. With annual benefit payments, the actual BP always equals 10,000 at 1/1.

Smith	birth date	01/01/23	spouse	01/01/26
	1/1/93 age	70		67

Case 1: Smith survives to 1/1/94, spouse dies

$$eAL_1 = 1.07(10{,}000\,\ddot{a}_{70} + P(\ddot{a}_{67} - \ddot{a}_{67:70})) - 1.07(10{,}000)$$
$$AL_1 = 10{,}000\,\ddot{a}_{71}$$
$$\text{Gain} = eAL_1 - AL_1 = 11{,}300$$

Case 2: Smith dies during 1993, spouse survives to 1/1/94

$$eAL_1 = \text{same as above}$$
$$AL_1 = P(\ddot{a}_{68})$$
$$\text{Gain} = eAL_1 - AL_1 = K$$

You can use the annuity factors to solve for the value of P under Case 1. Then you can substitute P and the annuities in Case 2 to solve for the value of K.

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(3) $11{,}300 = eAL_1 - AL_1$

$= 1.07(10{,}000(\ddot{a}_{70} - 1) + P(\ddot{a}_{67} - \ddot{a}_{67:\overline{70}})) - 10{,}000\,\ddot{a}_{71}$

$$P = \frac{(11{,}300 + 10{,}000\,\ddot{a}_{71})/1.07 - 10{,}000(\ddot{a}_{70} - 1)}{\ddot{a}_{67} - \ddot{a}_{67:70}}$$

$$= \frac{(11{,}300 + 73{,}680)/1.07 - 10{,}000(6.603)}{8.287 - 6.056}$$

$= 6{,}002.04$

$K = eAL_1 - AL_1$

$= 1.07(10{,}000(\ddot{a}_{70} - 1) + P(\ddot{a}_{67} - \ddot{a}_{67:70})) - P\ddot{a}_{68}$

$K - 11{,}300 = 10{,}000\,\ddot{a}_{71} - P\ddot{a}_{68}$

$K = 11{,}300 + 10{,}000(7.368) - 6002(8.061)$

$= 36{,}597.52$

D

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4 The experience gain is not calculated using the typical investment or non-investment G/L formulas. Instead, it is simply the difference at 1-1-93 between the accrued liability as an active employee, and the accrued liability as a retired employee.

One trick in this problem is the assumed retirement age of 62

	Active	Retired
1-1-93 Age	58	58
Entry Age	45	45
Past service	13	13
Total service	17	13
Retirement age	62	58
Early retirement reduction	.5%(12)(3) = 18%	.5%(12)(7) = 42%
Accrued benefit	12(10)(17) = 2040	12(10)(13) = 1560
Early retirement ben	(1-.18)2040 = 1672.80	(1-.42)1560 = 904.80
PVB at retirement	$1672.80\,\ddot{a}_{62}^{(12)}$ = 15,707.59	$904.80\,\ddot{a}_{58}^{(12)}$ = 9,247.06
PVB at entry age	$15{,}707.59(1.07)^{-17}$ = 4,973	
EANC	$4{,}973/\ddot{a}_{\overline{17}\rceil 7\%}$ = 476	
1-1-93 Accd liability	$476(\ddot{s}_{\overline{13}\rceil 7\%})$ = 10,258.07	

$\Delta = 10{,}258.07 - 9{,}247.06 = 1{,}011.01$

B

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5 The mortality loss can be calculated using the standard formula for non-investment gains and losses:

$$G/L = eAL_1 - AL_1$$
$$eAL_1 = (1+i)(NC_0 + AL_0) - (\text{actual } BP + i)$$

Since you are given the normal cost at 1-1-92, you can easily calculate the accrued liabilities at 1-1-92 and 1-1-93 on the retrospective definition. The key is that the entry age normal cost is still \$1,500 at 1-1-93

	1-1-92	1-1-93
Age	40	41
Entry age	30	30
Past service	10	11
Accrued liability	$NC(\ddot{s}_{30:\overline{10}\rceil})$	$NC(\ddot{s}_{30:\overline{11}\rceil})$
	$= NC\left(\frac{N_{30}-N_{40}}{D_{40}}\right)$	$= NC\left(\frac{N_{30}-N_{41}}{D_{41}}\right)$
	$= 1500\left(\frac{17{,}888-8{,}453}{632}\right)$	$= 1500\left(\frac{17{,}888-78\ldots}{590}\right)$
	$= 22{,}393$	$= 25{,}594$

$$N_{41} = N_{40} - D_{40} = 7{,}821$$

$$eAL_1 = 1.07(1{,}500 + 22{,}393) = 25{,}566$$
$$AL_1 = 25{,}594$$
$$\text{Loss} = 28$$

B

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6 Under the ILP cost method, each new layer of projected benefit is funded over future service. The 1-1-93 normal cost equals the 1-1-92 initial normal cost (calculated at plan inception) plus a new normal cost layer. The new layer funds the increase in projected benefit at 1-1-93 over future service at 1-1-93.

	1-1-92	1-1-93
Age	49	50
Entry age	40	40
Future service	16	15

Valuation compensation	16,000	18,000
Projected benefit	8,000	9,000
Δ in proj ben (PB)	8,000	1,000
PV of Δ (PB)	$8{,}000 v^{16} \ddot{a}_{65}^{(12)}$	$1{,}000 v^{15} \ddot{a}_{65}^{(12)}$
Δ normal cost	$\dfrac{8{,}000\, \ddot{a}_{65}^{(12)}}{\ddot{s}_{\overline{16}\rvert}\ 7\%}$ $= 2{,}343$	$\dfrac{1{,}000\, \ddot{a}_{65}^{(12)}}{\ddot{s}_{\overline{15}\rvert}\ 7\%}$ $= 325$

$\Sigma = 2{,}668$

C

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7 This ancillary benefit problem can be worked by carefully laying out all the required factors. The main complication is that you must derive the yearly decrements due to death and termination.

The general expression for the present value of death and termination benefits is

$$\sum_{x=61}^{65} v^{x-60} \left(\frac{l_x^{(T)}}{l_{61}^{(T)}}\right)\left(q_x^{(w)}+q_x^{(d)}\right) 15{,}000\,(1.06)^{x-60}$$

One thing to notice is that the interest rate and the interest credited on the 15,000 contributions cancel out!

Age x	$l_x^{(T)}/l_{61}^{(T)}$	$p_x^{(T)}$	$q_x^{(r)}$	$1-p_x^{(T)}-q_x^{(r)} = q_x^{(w)}+q_x^{(d)}$	$\left[\frac{l_x^{(T)}}{l_{61}^{(T)}}\right] * \left[q_x^{(w)}+q_x^{(d)}\right]$
61	1.0000	.9500	.0000	.0500	1.0000 (.0500) = .0500
62	.9500	.4895	.5000	.0105	.9500 (.0105) = .0100
63	.4650	.7742	.2000	.0258	.4650 (.0258) = .0120
64	.3600	.7722	.2000	.0278	.3600 (.0278) = .0100
65	.2780	—	1.0000	.0000	.2780 (.0000) = .0000
					.0820

PV = 15,000 (.0820) = 1230

B

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8 You must calculate the accrued liability on the old plan and compare the accrued liability on the new plan. Under the new plan there is a higher benefit and a more valuable present value factor.

The calculation of the weighted present value factor at age 65 is the key to the problem. The annuity value is 80% (married) times a 100% J+S annuity plus 20% (single) times a straight life annuity:

$$\text{New PV factor @ } 65 = .8\left(\ddot{a}_{65}^{(12)} + 1.0\left(\ddot{a}_{65}^{(12)} - \ddot{a}_{65:65}^{(12)}\right)\right) + .2\left(\ddot{a}_{65}^{(12)}\right)$$

$$= .8\left(\ddot{a}_{\overline{65:65}}^{(12)}\right) + .2\left(\ddot{a}_{65}^{(12)}\right)$$

$$= .8\,(10.576) + .2\,(8.736) = 10.208$$

Now you can calculate the AL under the old and new plan:

	OLD	NEW
1-1-93 Age	40	40
Entry age	27	27
Total service	38	38
Future service	25	25
# participants	100	100

(continued on next page)

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(8)

This is tricky! Different factors

	OLD PLAN	NEW PLAN
Age 40 pay	40,000	40,000
Age 64 pay	$40{,}000(1.03)^{24}$ $= 81{,}312$	$40{,}000(1.03)^{24}$ $= 81{,}312$
Final average factor	$\ddot{a}_{\overline{5}\rceil 3\%}/5$ $= .9434$	$\ddot{a}_{\overline{3}\rceil 3\%}/3$ $= .9712$
Final average pay at 64	76,711	78,966
Total PVB at 65	$100(.4)(76{,}711)\ddot{a}_{65}^{(12)}$ $= 26{,}805{,}933$	$100(.5)(78{,}966)(10.208$ $= 40{,}304{,}478$
PVB at entry age	$26{,}805{,}933(1.07)^{-38}$	$40{,}304{,}478(1.07)^{-38}$
Entry age normal cost	$26{,}805{,}933/\ddot{s}_{\overline{38}\rceil 7\%}$	$40{,}304{,}478/\ddot{s}_{\overline{38}\rceil 7\%}$
Accrued liability	$26{,}805{,}933\left(\dfrac{\ddot{s}_{\overline{13}\rceil 7\%}}{\ddot{s}_{\overline{38}\rceil 7\%}}\right)$ $= 3{,}128{,}683$	$40{,}304{,}478\left(\dfrac{\ddot{s}_{\overline{13}\rceil 7\%}}{\ddot{s}_{\overline{38}\rceil 7\%}}\right)$ $= 4{,}704{,}180$

$\Delta = 1{,}575{,}497$

(B)

Please note that our calculations really can't have more than 4 significant digits, since that is the accuracy implied in the annuity values at age 65.

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9 The first step is to write down formulas for the present value of benefits under each annuity:

I. $12(4000)\ddot{a}_{58}^{(12)}$

II. $12\left(3720\left[\ddot{a}_{58}^{(12)} + .5\left(\ddot{a}_{y}^{(12)} - \ddot{a}_{y:58}^{(12)}\right)\right]\right)$

III. $12\left(3500\,\ddot{a}_{58}^{(12)} + 500\left(\ddot{a}_{58}^{(12)} - \frac{D_{62}}{D_{58}}\ddot{a}_{62}^{(12)}\right) + K\left(\ddot{a}_{y}^{(12)} - \ddot{a}_{y:58}^{(12)}\right)\right)$

The second annuity is a 50% J+S annuity. The third annuity is a level income benefit which decreases by \$500 per month at age 62, but with an additional death benefit of \$K per month. By using the reversionary annuity concept, it should be relatively easy to write down the formulas shown above.

Since these annuities are actuarially equivalent, you can set them equal to each other. The key to the problem is to divide through by $\ddot{a}_{58}^{(12)}$:

I+II: $12(4000)\ddot{a}_{58}^{(12)} = 12\left(3720\left[\ddot{a}_{58}^{(12)} + .5\left(\ddot{a}_{y}^{(12)} - \ddot{a}_{y:58}^{(12)}\right)\right]\right)$

divide by $12\ddot{a}_{58}^{(12)}$: $4000 = 3720\left(1 + .5\left(\ddot{a}_{y}^{(12)} - \ddot{a}_{y:58}^{(12)}\right)/\ddot{a}_{58}^{(12)}\right)$

$$2\left(\frac{4000}{3720} - 1\right) = \frac{\ddot{a}_{y}^{(12)} - \ddot{a}_{y:58}^{(12)}}{\ddot{a}_{58}^{(12)}} = .1505$$

I+III: $12(4000)\ddot{a}_{58}^{(12)} = 12\left(3500\ddot{a}_{58}^{(12)} + 500\ddot{a}_{58}^{(12)} - 500\,N_{62}^{(12)}/N_{58}^{(12)} + K\left(\ddot{a}_{y}^{(12)} - \ddot{a}_{y:58}^{(12)}\right)\right)$

divide by $12\ddot{a}_{58}^{(12)}$: $4000 = 3500 + 500 - 500(.6867) + K(.1505)$

$$\frac{500(.6867)}{.1505} = K = 2{,}281$$

B

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10 The key to this problem is that the 1-1-93 entry age normal cost is exactly (30/15) times the 1-1-90 normal cost = 4000. It is unnecessary to try to solve for annuity functions or interest rates. You can calculate the 1-1-93 accrued liability using the retrospective definition:

1-1-93 Age 38
Entry age 35

$$
\begin{aligned}
\text{1-1-93 AL} &= (\text{EANC})\ \ddot{s}_{35:\overline{3}|} \\
&= 4000 \left(\frac{N_{35} - N_{38}}{D_{38}} \right) \\
&= 4000 \left(\frac{D_{35} + D_{36} + D_{37}}{D_{38}} \right) \\
&= 4000 \left(\frac{894 + 835 + 779}{727} \right) \\
&= 13{,}799
\end{aligned}
$$

C

This problem seems to be too easy. There is no other method of solution based on the information given.

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11 It is relatively easy to get the correct answer to this question. Even if you use the wrong approach (for individual cost methods) you may get the same answer!

Under an aggregate cost method, the liabilities of the plan include the present value of refunds of employee contribution. The assets of the plan include the present value of future employee contributions which is normally more valuable than the PV of future refunds:

$$PVB = 2{,}000{,}000 + 70{,}000 + 200{,}000$$
$$= 2{,}270{,}000$$

$$PVNC = PVB - AAV - PVEEC$$
$$= 2{,}270{,}000 - 500{,}000 - .015(7{,}000{,}000)$$
$$= 1{,}665{,}000$$

Note that the employee contributions were defined as payable at the beginning of the year, so the present value is correctly calculated as 1.5% times the PV of future pay.

$$PVE/E = 7{,}000{,}000 / 1{,}000{,}000 = 7.0$$
$$NC = 1{,}665{,}000 / 7.0$$
$$= 237{,}857$$

C

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12 This is an ancillary benefits problem. For a retirement benefit, exits are assumed to occur at the beginning of the year, which produces this general summation:

$$\Sigma\, v^t \, {}_tp_x \, q_{x+t}^{(r)} \, (ERB)_{x+t} \, \ddot{a}_{x+t}^{(12)}$$

The ERB_{x+t} term is the product of the accrued benefit (since we're using the unit credit cost method) and the early retirement reduction factor. Note that the last decrement age is 66, and that the postponed retirement benefit equals the normal retirement benefit.

The best way to work the problem is to write down several columns for the various terms in the summation:

t	Age $x+t$	(1) v^t	(2) $q_{x+t}^{(r)}$	(3) ${}_tp_x^{(T)}$	(4) ERB_{x+t}	(5) $\ddot{a}_{x+t}^{(12)}$	$(1)\times(2)\times(3)\times(4)\times(5)$ Product
4	64	.7629	.2	1.0	3600*(.94)	10.0	5,163
5	65	.7130	.5	(1-.2)=.8	3600	9.8	10,062
6	66	.6663	1.0	.8(1-.5)=.4	3600	9.6	9,212
							24,436

* The accrued benefit at 1-1-93 is calculated as \$15(12)(20) = 3600. The participant is age 60 at 1-1-93 with 20 years of past service.

A

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13 The experience G/L due to a change in compensation is not calculated using the "normal" formula. Instead, it is defined as the difference in the accrued liability calculated at 1-1-93 based on both expected and actual compensation.

One minor twist in this problem is that the entry age normal cost should be determined as a level dollar amount. If you had not been told to do that, usually you would calculate the normal cost as a level % of pay because the plan benefit is defined based on pay.

1-1-93 Age	45
Entry age	30
Past service	15
Total service	35
Actual age 45 pay	66,000
Projected age 64 pay	$66{,}000(1.05)^{19}$
= projected benefit	$=166{,}779$
PVB at entry age	$166{,}779\,(\ddot{a}_{65}^{(12)})(D_{65}/D_{30})$
	$=166{,}779\,(8.736)(1.07)^{-35}$
(Level Dollar) Entry age normal cost	$166{,}779\,(8.736)/\ddot{s}_{\overline{35}|.07}$
1-1-93 Accd liability	$166{,}779\,(8.736)(\ddot{s}_{\overline{15}|.07}/\ddot{s}_{\overline{35}|.07})$
	$=264{,}853$

Expected age 45 pay	$60{,}000(1.05)=63{,}000$
1-1-93 Expected Accd liability	$\left(\dfrac{63{,}000}{66{,}000}\right)264{,}853=252{,}814$
Δ = loss = 12,039	

D

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14 The accrued liability at 1-1-93 can be calculated using the retrospective definition:

$$AL_x = \Sigma (D_t / D_x)\, NC_t$$
$$= (1.07)(1992\,NC) + (1.07)^2 (1991\,NC)$$

Under the ILP method, each layer of projected benefit is funded over future service. The 1-1-92 normal cost equals the 1-1-91 initial normal cost <u>less</u> the change in normal cost to reflect the decrease in pay. The new layer funds the decrease in projected benefit over future service at 1-1-92.

	1-1-91	1-1-92
Age	51	52
Entry age	51	51
Future service	14	13
Valuation compensation	100,000	92,000
Projected benefit	50,000	46,000
Δ in Proj benefit	50,000	-4,000
PV of Δ in (PB)	$50{,}000\, \ddot{a}_{65}^{(12)} v^{14}$	$-4{,}000\, \ddot{a}_{65}^{(12)} v^{13}$
Δ Normal cost	$\dfrac{50{,}000\, \ddot{a}_{65}^{(12)}}{\ddot{s}_{\overline{14}\rvert 7\%}} = 18{,}111$	$\dfrac{-4{,}000\, \ddot{a}_{65}^{(12)}}{\ddot{s}_{\overline{13}\rvert 7\%}} = -1{,}622$
Total normal cost	18,111	16,489

$\therefore$ 1-1-93 Accrued liability $= 1.07(16{,}489) + (1.07)^2(18{,}111)$
$= 38{,}379$

C

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15 This problem requires you to develop an expected set of valuation results at 1-1-93, and then the actual valuation results. This is different than prior problems in that you can't easily calculate the normal cost as a percentage of pay. There is no way to demonstrate that ~~the~~ normal cost percentage is unchanged between the 1-1-92 valuation and the expected valuation results at 1-1-93.

	Actual 1-1-92	Expected 1-1-93	Actual 1-1-93
PVFB	1,200,000	1.07(1,200,000) − (BP+i)* = 1,284,000 − (BP+i)	1,284,000 (1.10/1.04) = 1,358,077
− UAL	UAL_0	$1.07(NC_0 + UAL_0 - 60{,}000)$	400,000
− AAV	500,000	1.07(500,000+60,000) − (BP+i)	599,200
= PVNC	$700{,}000 - UAL_0$	$1.07(1{,}200{,}000 - NC_0 - UAL_0 - 500{,}000)$	358,877
PVE	9,500,000	$1.07(9{,}500{,}000 - Earn_0)$ = 1.07 (9,500,000 − 750,000 ÷ 1.10) = 9,435,455	
Earn	750,000/1.10	750,000 (1.04/1.10) = 709,091	
PVE/E		13.3064 →	13.3064
Normal Cost			26,970

The key to solving the problem is that the PVE/E at 1-1-93 will not change if actives get a 10% pay increase. B

* assumed zero based on given data

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(15) As an exercise, I tried to solve for the UAL at 1-1-92 that would produce the desired UAL of 400,000 at 1-1-93. I started with a UAL of 400,000 and kept increasing it by the error in the 1-1-93 value:

	TRY #1	TRY #2	TRY #3	TRY #4	TRY #5
UAL_0	400,000	412,301	413,184	413,247	413,252
$PVNC_0$	300,000	287,699	286,816	286,753	286,748
$NCAR_0^*$	3.1579%	3.0284%	3.0191%	3.0185%	3.018400%
NC_0	21,531	20,648	20,585	20,580	20,580.00
UAL_1	386,838	399,055	399,933	399,995	400,000
$PVNC_1$	297,962	285,745	284,867	284,805	284,800
$NCAR_1^{**}$	3.1579%	3.0284%	3.0191%	3.0185%	3.018400%
ΔUAL	13,162	945	67	5	-0-

$$* \; NCAR_0 = \frac{PVNC_0}{PVE_0} = \frac{PVNC_0}{9{,}500{,}000}$$

$$** \; NCAR_1 = \frac{PVNC_1}{PVE_1} = \frac{PVNC_1}{9{,}435{,}455}$$

I found it interesting that the normal cost remained level as a percentage of payroll regardless of the initial value of the UAL! In other words, the normal cost as a percentage of payroll <u>must</u> be level, as long as you calculate the PVNC and PVE correctly. When I tried to solve for UAL_0 and NC_0 algebraically, they kept dropping out of both sides of the equation, or producing meaningless results.

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(15) Now the question in my mind is why <u>must</u> the NCAR be level if all assumptions are met. Logically we know it must be true, but why should it be true algebraically?

Given $\dfrac{700{,}000 - UAL_0}{9{,}500{,}000} = \dfrac{PVNC_0}{PVE_0} = \dfrac{NC_0}{681{,}818} = NCAR_0$

Show that $\dfrac{1.07(700{,}000 - UAL_0 - NC_0)}{1.07(9{,}500{,}000 - 681{,}818)} = \dfrac{PVNC_1}{PVE_1} = NCAR_1 = NCAR_0$

$$\frac{700{,}000 - UAL_0}{9{,}500{,}000}\left(\frac{9{,}500{,}000 - 681{,}818}{9{,}500{,}000 - 681{,}818}\right) = NCAR_0$$

$$\frac{(700{,}000 - UAL_0)(9{,}500{,}000 - 681{,}818)/9{,}500{,}000}{9{,}500{,}000 - 681{,}818} = NCAR_0$$

$$\frac{(700{,}000 - UAL_0)(1 - 681{,}818/9{,}500{,}000)}{9{,}500{,}000 - 681{,}818} = NCAR_0$$

From item Given, substitute 681,818/9,500,000

$$\frac{(700{,}000 - UAL_0)(1 - NC_0/(700{,}000 - UAL_0))}{9{,}500{,}000 - 681{,}818} = NCAR_0$$

$$\left(\frac{1.07}{1.07}\right)\frac{700{,}000 - UAL_0 - NC_0}{9{,}500{,}000 - 681{,}818} = NCAR_0$$

Q.E.D.

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16 The experience G/L can be calculated using the formula for non-investment gains and losses:

$$\text{non-inv } G/L = eAL_1 - AL_1$$

$$eAL_1 = (1+i)(NC_0 + AL_0) - (\text{actual } BP+i)$$

In this problem, the employee is vested, so the accrued liability at time 1 is not zero:

$$AL_1 = 5(12)(\$20)\, \ddot{a}_{65}^{(12)}\, D_{65}/D_{53}$$
$$= 1{,}200\,(8.74)(1.07)^{-12}$$
$$= 4{,}657$$

Now you must go back to 1-1-92 and determine last year's normal cost and accrued liability:

	1-1-92
Age	52
Entry age	48
Total service	17
Projected benefit	$4{,}080 = \$20(12)(17)$
PVB at entry age	$4{,}080\, \ddot{a}_{65}^{(12)}\, D_{65}/D_{48}$ $= 4{,}080\, \ddot{a}_{65}^{(12)}(1.07)^{-17}$
Entry age NC	$4{,}080\, \ddot{a}_{65}^{(12)} / \ddot{s}_{\overline{17}|.07}$ $= 1{,}081$
Accrued liability	$1{,}081\,(\ddot{s}_{\overline{4}|.07})$ $= 5{,}134$

$\therefore\ eAL_1 = 1.07(1{,}081 + 5{,}134) = 6{,}649$

$\therefore\ \text{gain} = 6{,}649 - 4{,}657 = 1{,}993$

B

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17 The attained age normal method is an aggregate cost method. The UAL is normally written down from one year to the next using the formula for the expected unfunded liability.

In this problem, you must use the normal cost to solve for the UAL. You can calculate the PVNC as the normal cost times a temporary annuity:

$$NC = \frac{PVNC}{\text{average annuity}} = \frac{PVNC}{\ddot{a}_{\overline{20|}.07}}$$

$$\therefore PVNC = NC \cdot \ddot{a}_{\overline{20|}.07}$$
$$= 1231(\ddot{a}_{\overline{20|}.07})$$
$$= 13{,}954$$

1-1-93 age 45
entry age 35
total service 30
future service 20

Projected ben $12(900)$
PV future ben $12(900)\ddot{a}_{65}^{(12)}(1.07)^{-20}$
$= 27{,}909$
$PVNC = PVB - UAL - AAV$
$13{,}954 = 27{,}909 - UAL - 10{,}000$
$\therefore \quad UAL = 17{,}909 - 13{,}954$
$= 3{,}955$ B

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18 Under the unit credit method, the accrued liability is equal to the present value of the accrued benefit.

	No mortality	With mortality
1-1-93 Age	53	53
Accrued liability	$AB \cdot \ddot{a}_{65}^{(12)} v^{65-x}$	$AB \cdot \ddot{a}_{65}^{(12)} \cdot \frac{D_{65}}{D_{53}}$
	$= AB \cdot \ddot{a}_{65}^{(12)} v^{12}$	$= AB \cdot \ddot{a}_{65}^{(12)} \frac{v^{65} l_{65}}{v^{53} l_{53}}$
		$= AB \cdot \ddot{a}_{65}^{(12)} v^{12} \left(\frac{l_{65}}{l_{53}}\right)$

The decrease in the accrued liability is the difference between these two values.

$$\Delta = AB \cdot \ddot{a}_{65}^{(12)} \cdot v^{12} \left[1 - \frac{l_{65}}{l_{53}}\right]$$

$$= 20{,}000 \left[1 - \frac{7{,}673{,}269}{8{,}980{,}994}\right]$$

$$= 2{,}912$$

E

Solution changed 2/12/200

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19 The key to working end of year valuation problems is making sure that the <u>number</u> of NC payments in the temporary annuity value is the same as it would be for a beginning of the year valuation problem

1-1-92 Age 52   Age 52 pay 50,000

#NC payments 13   Proj. Age 64 pay $50{,}000(1.03)^{12}$

$= 71{,}288$

Proj. Age 65 benefit $= 35{,}644 = .5(71{,}288)$

12-31-92 Age 53

$$PVB = 35{,}644\, \ddot{a}_{65}^{(12)}\, D_{65}/D_{53}$$
$$= 35{,}644\,(8.74)\,(1.07)^{-12}$$
$$= 138{,}322$$
$$PVNC = PVB - AAV = 138{,}322 - 10{,}000$$
$$= 128{,}322$$

The temporary annuity (with 13 payments!) with a salary scale of 3% must be calculated:

$$^{s}\ddot{a}_{53:\overline{13}|} = 1 + \left(\frac{1.03}{1.07}\right)^{1} + \dots + \left(\frac{1.03}{1.07}\right)^{12}$$
$$= \ddot{a}_{\overline{13}|j} \quad \text{where } 1+j = \frac{1.07}{1.03} = 1.0388$$
$$= \ddot{a}_{\overline{13}|3.88\%}$$
$$= 10.4488$$

$$\text{12-31-92 } NC = 128{,}322/10.4488$$
$$= 12{,}281$$

A

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20 This is an unusual problem. You have to set up the aggregate valuation results at both 1-1-92 and 1-1-93. Then you can derive the unit credit AL and UAL at 1-1-93.

1-1-92 Age 50
Entry age 45

1-1-92 NC = 898
$PVNC = 898\,(\ddot{a}_{\overline{15|}}\,8\%)$
$= 8{,}301$
$= PVB - AAV$
$\therefore\ PVB = PVNC + AAV$
$= 9{,}301$

With no noninvestment G/L, the sole participant survives to 1-1-93. With no preretirement deaths or terminations, the PVB grows with interest only from 1-1-92 to 1-1-93

1-1-93 Age 51
Entry age 45

1-1-93 $PVB = 1.08(9{,}301)$
$= 10{,}045$
$AAV = 1.05(1{,}000 + 898)$
$= 1{,}993$

The PVB is calculated based on a projected benefit with 20 years service, but the AL uses the accrued ben.

$AL = \frac{6}{20}(10{,}045) = 3{,}014$ $\quad UAL = 3{,}014 - 1{,}993$
$= 1{,}021$

E