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SPRING 1992 EA-1B EXAM SOLUTIONS

Revision History:

02/12/00	Corrected problem 10 – added alternate solution
02/12/00	Corrected problem 12 – added clearer solution
05/03/97	Corrected problem 04 - retrospective annuity on second page was 35:45, should be 35:10

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1 When the assumed retirement age changes, it will affect all of these items:

a) Projected benefit
b) Discount factor for PVB
c) Temporary annuity for NC calculation

Under the Aggregate cost method, PVNC = PVB − AAV. With one employee, the normal cost is calculated as PVNC / PVL when there is not a salary-related retirement benefit.

	After 1991 ARA 65	Before 1992 ARA 60	
1-1-92 Age	52	52	
Hire Age	40	40	
Total service	25	20	
Projected Benefit	7,500	6,000	payable at ARA
PV factor	$v^{13}\ddot{a}_{66}^{(12)}$	$v^{8}\ddot{a}_{60}^{(12)}$	
	$=(1.07)^{-13}(8.74)$	$(1.07)^{-8}(9.82)$	
	$=$ 3.6268	5.7153	
PVB at 1-1-92	27,201	34,292	
PVNC	22,201	29,292	
$\ddot{a}_{x:\overline{RA-x}\rceil}$	$\ddot{a}_{\overline{13}\rceil .07}$	$\ddot{a}_{\overline{8}\rceil .07}$	(no decrements)
	$=$ 8.9427	$=$ 6.3893	
Normal cost	2,483	4,585	

Decrease in normal cost due to change in assumed retirement age from 60 to 65 is 2483 − 4585 = (2102).

D

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2 Under the Aggregate cost method, the PVNC equals PVB − AAV. The normal cost will be a level percent of payroll as long as all assumptions are met, and if the normal cost only is paid as of the valuation date.

<u>1-1-91 valuation</u>

No payroll given, assume value of 100,000

NC = 7.75% of pay

PVNC = 7750 (20.13) = 156,008

AAV = ∅ since 1-1-91 is plan effective date

PVB = 156,008

<u>1-1-92 valuation - expected results</u>

Expected asset value = 1.07 (7,750) = 8292. Since this matches the given assets, and there are no pre-retirement decrements, the only term that will affect the final valuation results is the 8% pay increase.

Expected pay = 105,000 = 1.05 (100,000)

Expected NC = 105,000 (7.75%) = 8,138 = 1.05 (7,750)

Expected PVE = 1.07 (20.13 (100,000) − 100,000)

= 20,469,100

PVE/E = 20,469,100 / 105,000 = 19.49

Expected PVNC = 19.49 (8,138) = 158,636

Expected PVB = 158,636 + 8,292 = 166,928

<u>1-1-92 valuation - actual</u>

Actual PVB = (1.08/1.05) 166,928 = 171,697

PVNC = 171,697 − 8,292 = 163,405

PVE/E = 19.49 (same as expected)

NC = 8,384

D

Δ NC = 8,384 − 8,138 = 246

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3 With early retirement decrements, you can express the PVB using a summation (from ancillary benefits):

$$PVB = \sum_{t=0}^{3} v^t \, {}_tp_{62} \, q_{62+t}^{(r)} \, ERB_{62+t} \, \ddot{a}_{62+t}^{(12)}$$

Note that ERB_{62+t} is the reduced early retirement benefit, and we're using v^t since retirement is assumed to occur at the beginning of each year.

The best way to work the problem is to set up a large table that allows you to calculate each of the terms in the summation, with each retirement age on a separate row:

t	Age $62+t$	(1) ${}_tp_{62}$	(2) $q_{62+t}^{(r)}$	Accd Ben at $62+t$	(3) ERB_{62+t}	(4) $v^t \ddot{a}_{62+t}^{(12)}$	$v^t {}_tp_{62} q_{62+t}^{(r)} ERB_{62+t} \ddot{a}_{62+t}^{(12)}$ (1)x(2)x(3)x(4)
0	62	1.000	.25	240(12)	2880(12/15)	(1.00) 9.18	5288
1	63	.750	.50	240(13)	3120(13/15)	(.9346) 8.96	8491
2	64	.375	.75	240(14)	3360(14/15)	(.8734) 8.74	6733
3	65	.09375	1.00	240(15)	3600(15/15)	(.8163) 8.51	2345
							22856

The key point is that ${}_tp_{62} = {}_{t-1}p_{62}(1-q_{62+t-1}^{(r)})$. Many people have a hard time believing that only 10% of the original population is left at age 65, but it's true!

B

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4 Under the Entry Age Normal cost method, the normal cost equals PVB_{EA}/PVX_{EA}. If the benefit is pay-related, the $EANC\% = PVB_{EA}/PVE_{EA}$. The accrued liability is either the accumulated value of past normal costs (with salary scale, interest and survivorship), or it can be defined as PVB − PVNC.

$$eAL_1 = (1+i)(NC_0 + AL_0) - (\text{actual } BP + i)$$

This problem is mostly messy arithmetic to calculate the NC and AL at 1-1-92:

$$EANC\% = PVB_{EA} / PVE_{EA}$$

1-1-92 Age 45
Entry Age 35
Total Svc 30

1991 pay at age 44 = 28,571
pay at age 64 = $28{,}571(1.05)^{20} = 75{,}807$
FAE_3 at age 64 = $75{,}807(\ddot{a}_{\overline{3}|}/3) = 72{,}255$

Projected benefit $= .5(72{,}255) = 36{,}127$

$$PVB_{EA} = 36{,}127\ \ddot{a}_{65}^{(12)}\ D_{65}/D_{35}$$
$$= 36{,}127\ (8.74)(94/894) = 33{,}200$$

Pay at entry age 35 $= 28{,}571(1.05)^{-9} = 18{,}417$

$$PVE_{EA} = 18{,}417({}^{s}\ddot{a}_{35:\overline{30}|}) = 18{,}417\left(\frac{{}^{s}N_{35} - {}^{s}N_{65}}{{}^{s}D_{35}}\right)$$
$$= 18{,}417\ (138{,}500 - 30{,}014)/4{,}931$$
$$= 405{,}192$$

$$EANC\% = 33{,}200/405{,}192$$
$$= 8.1937\%$$

Revised 5/3/97

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(4) Continued

1-1-92 EANC = (1992 pay)(EANC%)
= 1.05 (28,571) (8.1937%)
= 2,458

Retrospective AL = $2{,}458\left({}^{s}\ddot{s}_{35:\overline{10|}}\right) = 2{,}458\left(\dfrac{{}^{s}N_{35} - {}^{s}N_{45}}{{}^{s}D_{45}}\right)$
= 2,458 (138,500 − 93,473)/3,998
= 27,684

Should check the prospective definition of the AL to eliminate possibility of arithmetic errors.

Prospective AL = PVB − PVNC
$= 36{,}127(8.74)(94/445) - 2{,}458\,\dfrac{(93{,}473 - 39{,}0\ldots)}{3{,}998}$

= 66,698 − 39,016
= 27,682 OK, close enough

$eAL_1 = 1.07(27{,}684 + 2{,}458)$
= 32,251

(B)

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5 Under the Entry Age Normal cost method, the normal cost is defined as $PVB_{EA}/PV\cancel{B}_{EA}$. With a benefit defined as dollar per month, the normal cost should be calculated as PVB_{EA}/PVL_{EA}.

The change in retirement age will change the projected benefit as well as the discounted PV at entry age, and the temporary annuity from entry age to retirement age.

1-1-92 Age 57

	Before 1992 ARA 62	After 1991 ARA 65	
Entry age	45	45	
Total svc	17	20	
Proj Benefit	17(120)	20(120)	note: unreduced
	= 2040	= 2400	early retire
PVB_{EA}	$2040\,\ddot{a}_{62}^{(12)}\,D_{62}/D_{45}$	$2400\,\ddot{a}_{65}^{(12)}\,D_{65}/D_{45}$	
	$=2040(9.39)(1.07)^{-17}$	$=2400(8.74)(1.07)^{-20}$	
PVL_{EA}	$\ddot{a}_{45:\overline{17}\vert}$	$\ddot{a}_{45:\overline{20}\vert}$	
	$=\ddot{a}_{\overline{17}\vert 7\%}$	$=\ddot{a}_{\overline{20}\vert 7\%}$	
EANC	$2040(9.39)/\ddot{s}_{\overline{17}\vert 7\%}$	$2400(8.74)/\ddot{s}_{\overline{20}\vert 7\%}$	
AL	$EANC\ \ddot{s}_{\overline{12}\vert 7\%}$	$EANC\ \ddot{s}_{\overline{12}\vert 7\%}$	
	$=19156(\ddot{s}_{\overline{12}\vert}/\ddot{s}_{\overline{17}\vert})$	$=20{,}976(\ddot{s}_{\overline{12}\vert}/\ddot{s}_{\overline{20}\vert})$	
	= 11,111	= 9,153	

$\Delta = (1958)$

B

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6 Under the Individual Level Premium cost method, each change in projected benefit is funded prospectively over future service. The ~~total~~ ILP normal cost equals the sum of this year's new layer and last years normal cost.

$G/L = eAL_1 - AL_1$

1-1-92 Age 55
Past svc 20 years

Acc'd benefit $= 12(41.67)20 = 10{,}001$
PV of acc'd $= 10{,}001\, \ddot{a}_{65}^{(12)} D_{65}/D_{55}$
$= 10{,}001(8.51)(1.07)^{-10}(.8562)$
$= 37{,}043$

Since the early retirement benefit is reduced to the actuarial equivalent, the retirement annuity is also worth $37{,}043 = ERB(\ddot{a}_{55}^{(12)}) = AL_1$.

Hire Age 35
Total svc 30 years

Projected ben $= 12(41.67)30 = 15{,}001$
$PVB = 15{,}001\, \ddot{a}_{65}^{(12)} D_{65}/D_{55}$
$= (15{,}001/10{,}001)\, 37{,}043$
$= 55565$

1-1-82 Age 45
1-1-82 ILP NC $= \dfrac{15{,}001\, \ddot{a}_{65}^{(12)} D_{65}/D_{45}}{\ddot{a}_{45:\overline{20|}}}$

$$AL = PVB - PVNC = 15{,}001\, \ddot{a}_{65}^{(12)} \frac{D_{65}}{D_{55}} - \frac{15{,}001\, \ddot{a}_{65}^{(12)} D_{65}}{D_{45}(\ddot{a}_{45:\overline{20|}})}\left(\ddot{a}_{55:\overline{10|}}\right)$$

$$= 15{,}001\, \ddot{a}_{65}^{(12)} \frac{D_{65}}{D_{55}}\left[1 - \frac{\ddot{a}_{55:\overline{10|}}\, D_{55}}{\ddot{a}_{45:\overline{20|}}\, D_{45}}\right] = 55{,}565\left[1 - \frac{N_{55}-N_{65}}{N_{45}-N_{65}}\right]$$

$$= 55{,}565\left[\frac{N_{45}-N_{55}}{N_{45}-N_{65}}\right] = 55{,}565\left(\frac{\ddot{a}_{45:\overline{10|}}}{\ddot{a}_{45:\overline{20|}}}\right)$$

$= 55{,}565\,(7.37/10.79) = 37{,}953$ $\quad \Delta AL = 910$

A

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7 With a split funded pension plan, there are several steps which should be followed to calculate the side fund normal cost:
1. Calculate accrued or projected benefit
2. Determine face amount of life insurance
3. Calculate cash surrender value at assumed ret age
4. Convert cash surrender value to benefit
5. Calculate ~~total~~ projected benefit net of item 4
6. Apply cost method to net benefit in step 5

1-1-92 Age 40 Hire age 35 Total service 30 years

Projected benefit $30(12)(40) = 14{,}400$
Insurance $= 50(30)(40) = \underline{60{,}000}$
CSV at 65 $= 60{,}000\ (200/1{,}000) = 12{,}000$
Insurance provided benefit at 65 $= 12{,}000/8.74$
$= 1{,}373$
Net benefit at 65 $= 14{,}400 - 1{,}373 = 13{,}027$

1-1-92 PVFB $= 13{,}027 \left(\ddot{a}_{65}^{(12)} D_{65}/D_{40}\right)$
$= 13{,}027\,(8.74)\,(1.07)^{-25}$
$= 20{,}978$
PVNC $=$ PVFB $-$ AAV $= 15{,}978$
PVL $= \ddot{a}_{\overline{25}|7\%} = 12.4693$
Side Fund NC $= 15{,}978 / 12.4693 = 1{,}281$

The annual premium would be used if the problem asked for the total normal cost, which is the sum of the side fund NC and the insurance premium.

B

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8 Under the Individual Level Premium cost method, each change in projected benefits (including those due to salary changes) produces a new layer of normal cost. The normal cost is always funded over future service, so an increase in benefits does not increase the accrued liability:

$$\Delta NC = \frac{\Delta PVB}{\ddot{a}_{x:\overline{RA-x}|}} \quad \text{or} \quad \Delta NC\% = \frac{\Delta PVB}{PVE}$$

Since there is no salary scale in this problem, we can calculate the normal cost layer as a constant dollar amount.

1-1-91 Age 40 Projected benefit $60{,}000(.5) = 30{,}000$

$$\text{1-1-91 NC} = \frac{30{,}000\, \ddot{a}^{(12)}_{65}\, D_{65}/D_{40}}{\ddot{a}_{40:\overline{25}|}} = \frac{30{,}000\,(8.74)}{\ddot{s}_{\overline{25}|\,7\%}}$$

$$= 3{,}874$$

1-1-92 Age 41

$$\text{1-1-92 } \Delta NC = 3032 - 3874 = -842$$

$$= \frac{\text{"}\Delta PVB\text{"}}{\ddot{s}_{\overline{24}|\,7\%}} = \frac{(\text{Change in 1992 Pay}).5\, \ddot{a}^{(12)}_{65}}{62.2490}$$

$$\therefore \text{Change in 1992 Pay} = \frac{-842\,(62.249)}{.5\,(8.74)}$$

$$= -11{,}998$$

$$\therefore \text{1992 Pay} = 60{,}000 - 11{,}998 = 48{,}002$$

B

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9 Under the Unit Credit cost method, if the early retirement benefit is actuarially reduced, then there is no experience gain or loss:

AL as active = (Accd Benefit) $N_{65}^{(12)}/D_x$
AL as retiree = (Early Ret Ben) $N_x^{(12)}/D_x$

If Early Ret Ben = Accd Benefit $(N_{65}^{(12)}/N_x^{(12)})$, then there is no gain or loss on early retirement.

$$\frac{N_{65}^{(12)}}{N_x^{(12)}} = \frac{D_{65}\ddot{a}_{65}^{(12)}}{D_x\ddot{a}_x^{(12)}} \Rightarrow \text{no pre-retirement decrement} \Rightarrow (1.06)^{x-65}\frac{\ddot{a}_{65}^{(12)}}{\ddot{a}_x^{(12)}}$$

Age	Plan Reduction		Actuarial Equivalent	
56	$1-5/15-4/30$	$= .5333$	$(1.06)^{-9}\,10/11.50 = .5147$	Loss
59	$1-5/15-1/30$	$= .6333$	$(1.06)^{-6}\,10/11.00 = .6409$	Gain
63	$1-2/15$	$= .8667$	$(1.06)^{-2}\,10/10.33 = .8616$	Loss

Of the three retirement ages, only retirement at age 59 produces a lower benefit than an actuarial equivalent.

(A)

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10 In general, the experience G/L can be calculated as $eAL_1 - AL_1$.

$eAL_1 = (1+i)(NC_0 + AL_0) -$ (actual BP+i)

Since these retirees were in pay status the NC_0 term is zero ↙ SNOPAKE

$$eAL_1 = 1.07(100)(10{,}000)\left(\ddot{a}_{70}^{(12)} + .5\left(\ddot{a}_{67}^{(12)} - \ddot{a}_{70:67}^{(12)}\right)\right)$$
$$- 1.07(100)(10{,}000)$$
$$= 1{,}070{,}000\,(8.06 + .5(8.74 - 6.51) - 1)$$
$$= 8{,}747{,}250$$

$$AL_1 = 93(10{,}000)\left(\ddot{a}_{71} + .5\left(\ddot{a}_{68} - \ddot{a}_{71:68}\right)\right) \quad \text{both alive}$$
$$+ 2(10{,}000)(\ddot{a}_{71}) \quad \text{retiree alive}$$
$$+ 3(5{,}000)(\ddot{a}_{68}) \quad \text{spouse alive}$$
$$+ 2(0) \quad \text{both dead}$$
$$= 930{,}000(7.83 + .5(8.52 - \ddot{a}_{71:68})) + 20{,}000(7.83) + 15{,}000(8.52)$$

You must solve for the value of $\ddot{a}_{71:68}$. Since

$$\ddot{a}_{xy} = 1 + vp_{xy} + v^2\,{}_2p_{xy} + \ldots$$
$$\ddot{a}_{x+1:y+1} = 1 + vp_{x+1:y+1} + v^2\,{}_2p_{x+1:y+1} + \ldots$$
$$1 + vp_{xy}\,\ddot{a}_{x+1:y+1} = \ddot{a}_{xy}$$
$$\therefore \ddot{a}_{x+1:y+1} = \frac{(\ddot{a}_{xy} - 1)(1+i)}{p_{xy}} \qquad \ddot{a}_{71:68} = \frac{(\ddot{a}_{70:67} - 1)(1.07)}{p_{70:67}}$$

$p_{xy} = p_x(p_y)$

You are given values of $\ddot{a}_x$, $\ddot{a}_{x+1}$, $\ddot{a}_y$ and $\ddot{a}_{y+1}$, and you can solve for the values of p_x and p_y:

$$1 + vp_x\,\ddot{a}_{x+1} = \ddot{a}_x \Rightarrow p_x = \frac{(\ddot{a}_x - 1)(1+i)}{\ddot{a}_{x+1}}$$

$p_{70} = (7.06)(1.07)/7.83 = .9648 \quad p_{67} = (7.74)(1.07)/8.52 = .9720$

$p_{70:67} = .9378 \quad \ddot{a}_{71:68} = 6.2867$

Solution added 2/12/2000

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10 Continued

There is a slightly different method of solution that gives a similar result. Instead of the initial general formula for the expected accrued liability, you can construct it based on all possible cases of survival:

$$
\begin{aligned}
eAL_1 &= 100(p_{70}p_{67})(10{,}000)(\ddot{a}_{71} + .50(\ddot{a}_{68} - \ddot{a}_{71:68})) && \text{both alive} \\
&+ 100(p_{70}q_{67})(10{,}000)\,\ddot{a}_{71} && \text{Partic alive} \\
&+ 100(q_{70}p_{67})(5{,}000)\,\ddot{a}_{68} && \text{Spouse alive} \\
&+ 100(q_{70}q_{67})(0) && \text{Both dead} \\
&= 100(.9648)(.9720)(10{,}000)(7.83 + .5(8.52 - 6.2867)) \\
&\quad + 100(.9648)(1-.9720)(10{,}000)(7.83) \\
&\quad + 100(1-.9648)(.9720)(5{,}000)(8.52) \\
&= 8{,}747{,}404
\end{aligned}
$$

$eAL_1 - AL_1 = \text{Gain} = 144{,}149$

As described earlier, this produces a slightly different answer. Since the values only have three significant digits for the answer range, it is still in range D.

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11 Under the Aggregate cost method, the PVNC is equal to PVB - AAV. The only trick in this problem is that the 7% pay increase will not affect the PVB for non-active participants.

The first step is to determine the PVNC and NC for the original valuation results. The PVE/E factor will be the same for the revised valuation, since both items would be 7% greater.

	Initial valuation	7% pay increase
PVB-active	1,280,000	1,369,600 = 1.07(1,280,000)
PVB - inactive	320,000	320,000
Total PVB	1,600,000	1,689,600
AAV	400,000	400,000
PVNC = PVB - AAV	1,200,000	1,289,600
PVE/E	19.2 / 3.2 = 6.0	6.0
Normal cost	200,000	214,933

C

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12 Since the optional forms of benefit payment are actuarially equivalent, the present value of each benefit equals the present value of the 1000 monthly life annuity with a five year certain period.

Since you are not given any interest rates or present value factors, this is simply an algebra problem. You should be able to express the present values so that all the annuity factors drop out

$(12)1000\left(\ddot{a}^{(12)}_{\overline{5}|} + N^{(12)}_{x+5}/D_x\right)$ = PV of normal form

$(12)\ 840\left(\ddot{a}^{(12)}_{\overline{5}|} + N^{(12)}_{x+5}/D_x + .5\left(N^{(12)}_{y+5}/D_y - N^{(12)}_{x+5:y+5}/D_{xy}\right)\right)$
is the present value of optional form A

$(12)\ (B)\left(\ddot{a}^{(12)}_{\overline{5}|} + N^{(12)}_{x+5}/D_x + .75\left(N^{(12)}_{y+5}/D_y - N^{(12)}_{x+5:y+5}/D_{xy}\right)\right)$
is the present value of optional form B

All of the above items are equal. By dividing the present values of A and B by the benefit amounts and multiplying the resulting PV for A by 1.5, the factors for the reversionary annuity will match B:

$\frac{1000}{840}\left(\ddot{a}^{(12)}_{\overline{5}|} + N^{(12)}_{x+5}/D_x\right) = \ddot{a}^{(12)}_{\overline{x:5}|} + .5\left({}_{5|}\ddot{a}^{(12)}_y - {}_{5|}\ddot{a}^{(12)}_{xy}\right)$

$(1500/840)\ \ddot{a}^{(12)}_{\overline{x:5}|} = 1.5\ \ddot{a}^{(12)}_{\overline{x:5}|} + .75\left({}_{5|}\ddot{a}^{(12)}_y - {}_{5|}\ddot{a}^{(12)}_{xy}\right)$

$(1000/B)\ \ddot{a}^{(12)}_{\overline{x:5}|} = \ddot{a}^{(12)}_{\overline{x:5}|} + .75\left({}_{5|}\ddot{a}^{(12)}_y - {}_{5|}\ddot{a}^{(12)}_{xy}\right)$

$(1500/840 - 1000/B)\ \ddot{a}^{(12)}_{\overline{x:5}|} = .5\ \ddot{a}^{(12)}_{\overline{x:5}|}$

$1.7857 - 1000/B = .5$ (divided by $\ddot{a}^{(12)}_{\overline{x:5}|}$)

D

$1.2857 = 1000/B$

$B = 777.78$

(12) Continued

A cleaner way to solve the algebra problem is to divide both sides by $\ddot{a}^{(12)}_{\overline{5}|} + N^{(12)}_{x+5}/D_x$:

Optional Form A

$$1000\left(\ddot{a}^{(12)}_{\overline{5}|} + N^{(12)}_{x+5}/D_x\right) = 840\left[\ddot{a}^{(12)}_{\overline{5}|} + N^{(12)}_{x+5}/D_x + .5\left(\frac{N^{(12)}_{y+5}}{D_y} - \frac{N^{(12)}_{x+5:y+5}}{D_{xy}}\right)\right]$$

$1000 = 840\,(1 + .5R)$ where R is the ratio of the five year deferred J+S annuity divided by $\ddot{a}^{(12)}_{\overline{5}|} + N^{(12)}_{x+5}/D_x$

Optional Form B

$$1000\left(\ddot{a}^{(12)}_{\overline{5}|} + \frac{N^{(12)}_{x+5}}{D_x}\right) = B\left[\ddot{a}^{(12)}_{\overline{5}|} + \frac{N^{(12)}_{x+5}}{D_x} + .75\left(\frac{N^{(12)}_{y+5}}{D_y} - \frac{N^{(12)}_{x+5:y+5}}{D_{xy}}\right)\right]$$

$$1000 = B\,(1 + .75R)$$

Now you have two equations in two unknowns. Solve for the value of R in the first equation:

$$\frac{1000}{840} = 1 + .5R = 1.1905$$

$$.5R = .1905$$

$$R = .381$$

Now substitute the value of R into the second equation:

$$1000 = B(1 + .75(.381))$$

$$B = \frac{1000}{1.2857} = 777.78$$

D

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13 This problem has two methods of solution, one of which is much longer! The short method is to perceive this as a G/L question. Brown is retired at 1-1-92, and Smith is active. Under the Frozen Initial Liability method, the normal cost would be unchanged at 1-1-93 if all assumptions are met. Since Brown is alive at 1-1-93, there has been a mortality loss that will increase the normal cost. The assets earned the expected rate of interest.

1-1-92 PVNC = PVB − AAV − UAL

$= 10{,}841 + 41{,}952 - 41{,}952 - 10{,}000$

$= 841$

avg PVL $= \ddot{a}_{20\rceil .07}$ (Smith is age 45)

$= 11.3356$

NC $= 74.19$

1-1-93 expected PVNC $= 74.19\,(\ddot{a}_{19\rceil .07}) = 820.49$

expected NC $= 74.19$

G/L = $eAL_1 - AL_1$ for Brown

$AL_1 = \left(\frac{41{,}952}{8.74}\right) 8.51 = 4800\,(8.51) = 40{,}848$

$eAL_1 = (1+i)(NC_0 + AL_0) - $ (actual BP+i)

$= 1.07\,(0 + 41{,}952) - 4800\left(1 + \frac{.07}{12}\left(\frac{12}{12}\right) + \frac{.07}{12}\left(\frac{11}{12}\right) + \dots + \frac{.07}{12}\left(\frac{1}{12}\right)\right)$

$= 44{,}889 - 4800\left(1 + \frac{13}{24}(.07)\right)$

$= 39{,}907$

Loss $= 941 = 40{,}848 - 39{,}907$

1-1-93 actual PVNC $= 941 + 820 = 1761$ NC $= 1761/\ddot{a}_{19\rceil .07} = 159$ C

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(13) Continued

The alternative approach is to treat this as a routine Frozen Initial Liability problem. This requires you to separately calculate the values at 1-1-93 of AAV, PVB and UAL.

Brown's benefit $= 41{,}952/8.74 = 4800$

$$\text{1-1-93 AAV} = 1.07(41{,}952) + 1.07(NC_0 + 10{,}000/\ddot{a}_{\overline{10}|.07}) - 4800\left(1 + \frac{.07}{12}\left[\frac{12}{12} + \frac{11}{12} + \ldots + \frac{1}{12}\right]\right)$$

$$\text{1-1-92 PVNC} = PVB - AAV - UAL$$
$$= 10{,}841 + 41{,}952 - 41{,}952 - 10{,}000$$
$$= 841$$
$$NC = 841/\ddot{a}_{\overline{20}|.07} = 74.19$$

$$\text{1-1-93 AAV} = 1.07(41{,}952 + 74.19 + 1331) - 4800\left(1 + \frac{.07}{12}\left(\frac{12(13)}{12(2)}\right)\right)$$

$$= 41{,}410 \quad = 46{,}392 - 4{,}982$$

$$\text{1-1-93 UAL} = eUAL = (1+i)(NC_0 + UAL_0) - (\text{Contrib} + i)$$
$$= 1.07(74.19 + 10{,}000) - 1.07(74.19 + 1331)$$
$$= 9{,}276 \quad = 1.07(10{,}000 - 1331)$$

1-1-93 PVB for Smith $= 1.07(10{,}841) = 11{,}600$
for Brown $= 4800(8.51) = \underline{40{,}848}$
$52{,}448$

$$\text{1-1-93 PVNC} = 52{,}448 - 9{,}276 - 41{,}410$$
$$= 1{,}762$$

avg PVL $= \ddot{a}_{\overline{19}|.07}$ $\quad NC = 1762/\ddot{a}_{\overline{19}|.07} = 159$

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14 Under the Unit Credit cost method, the accrued liability is defined equal to the present value of the accrued benefit. The normal cost is then equal to the present value of the change in the accrued benefit.

Under the term cost method, the normal cost would equal the present value of benefits for exits for one year. That is the definition of "term cost", the cost for one year's terminations.

<u>1-1-92 valuation</u>

Age 45	Projected monthly benefit $= 40(\$10) = 400$
Hire age 25	Pre-retirement death ben $= 40{,}000$
Total svc 40 years	PV for deaths at 45 $= v\, q^{d}_{45}(40{,}000)$

$$\frac{D_{45}}{D_{46}} = \frac{1+i}{p_{45}} \Rightarrow p_{45} = (1+i)\frac{D_{46}}{D_{45}} \Rightarrow q^{d}_{45} = 1 - (1+i)\frac{D_{46}}{D_{45}}$$

$$= 1 - 1.07(4236/4548)$$

$$= .0034$$

$$\text{PV for deaths at } 45 = \frac{.0034}{1.07}(40{,}000) = 127.24$$

$$\text{Unit Credit normal cost} = (\Delta\,\text{Accd Ben})\,(N^{(12)}_{65} / D_x)$$

$$= 120\left(\frac{N_{65} - \frac{11}{24}D_{65}}{D_{45}}\right)$$

$$= 120\,(8872 - \tfrac{11}{24}(965))/4548$$

$$= 222.42$$

Total normal cost $= 127.24 + 222.42 = 349.66$

D

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15 Under the Unit Credit cost method, the accrued liability is defined as the present value of the accrued benefit. Under the Entry Age Normal method, the normal cost is defined as PVB_{EA}/PVX_{EA}. The accrued liability can be calculated as the accumulated value of past normal costs.

1-1-92 Age 50   Past service 5 years
Hire age 45   Total service 20 years

pre-'92 **Unit Credit valuation results**

$$\text{Accrued liability} = (\text{Accd Benefit})(N_{65}^{(12)}/D_x)$$
$$= 5(12)(\$50)\ (8.74)\ D_{65}/D_x$$
$$= 3{,}000\ (8.74)(1.07)^{-15}$$
$$= 9{,}503$$

post-'91 **Entry Age Normal valuation results**

$$EANC = PVB_{EA}/PVL_{EA}$$
$$PVB_{EA} = (\text{Projected Benefit})(\ddot{a}_{65}^{(12)})(D_{65}/D_{EA})$$
$$= 20(12)(\$50)\ \ddot{a}_{65}^{(12)}\ v^{20}$$
$$PVL_{EA} = \ddot{a}_{\overline{20}|.07}$$
$$EANC = 12{,}000(8.74)/43.8652$$
$$= 2{,}391$$

$$\text{Accd Liability} = 2{,}391\ \ddot{s}_{\overline{5}|.07} = 14{,}712$$

$$\text{Increase in accrued liability} = 14{,}712 - 9{,}503 = 5{,}209$$

D

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16 Under the Projected Unit Credit cost method, calculation of normal cost and accrued liability are similar to the Unit Credit method. The only difference is that benefits are calculated including salary projections to assumed retirement.

1-1-92 Age 55
Hire Age 50
Past Service 5 years

1991 pay 31,000 corresponds to age 54
$(1.04)^{10}(31,000)$ corresponds to age 64
$= 45,888$

3 year FAE at age 65 $= 45,888\,(\ddot{a}_{\overline{3}|.04}/3) = 44,145$

1-1-92 F.A.B. $= .02(5)(44,145)$
1-1-93 F.A.B. $= .02(6)(44,145)$
1-1-92 Δ(F.A.B.) $= .02(1)(44,145)$
1-1-92 NC $= (\Delta FAB)\,\ddot{a}_{65}^{(12)}\, D_{65}/D_x$
$= .02(44,145)(8.50)(1.08)^{-10}$
$= 3,476$

E

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17 Under any cost method, the gain or loss must be calculated using the same assumptions and benefits as the previous valuation. This way, the eUAL and UAL are calculated on a consistent basis, and the difference between them is only caused by experience gains and losses.

Under the Unit Credit cost method, the accrued liability is defined as the present value of the accrued benefit. The experience G/L is the difference between the expected and actual unfunded accrued liability.

1-1-92 $eUAL = (1+i)(NC_0 + UAL_0) - (\text{contrib} + i)$
$= 1.07(1{,}000 + 10{,}000) - 2{,}000(1.035)$
$= 9{,}700$

1-1-92 valuation results

\$10.50 ben	\$10.00 ben	
22,500	(10/10.5) 22,500 = 21,429	Accrued liability

1-1-92 UAL on \$10.00 benefit = 21,429 − 13,500 = 7,929

G/L = 9,700 − 7,929 = 1,771

This is a gain, since the actual UAL is less than what we expected at 1-1-92.

D

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18 You can use the typical expression for the present value of ancillary benefits. One change is the v^t term (instead of v^{t+1}), which is due to the assumption of death at the beginning of the year:

$$PV\ \text{death benefits} = \sum_{t=0}^{64-t} v^t\ {}_tp_x\ q^d_{x+t}\ DB_{x+t}\ PVF_{x+t}$$

$$= \sum_{t=0}^{2} v^t\ {}_tp_{62}\ q^d_{62+t}\ DB_{x+t}\ (1.0)$$

The participant is 62 at 1-1-92. The death benefit varies based on the accrued benefit each year. The following table builds the pieces of each term in the summation:

t	Age $x+t$	(1) v^t	(2) ${}_tp_{62}$	(3) q^d_{62+t}	(4) SVC_{x+t}	(5) Monthly $Accd_{x+t}$	(6) DB_{x+t}	(7) = (1)(2)(3)(6) $v^t\ {}_tp_{62}\ q^d_{62+t}\ DB_{x+t}$
0	62	1	1.0000	.017	32	480	48,000	816
1	63	.9346	.9830	.019	33	495	49,500	864
2	64	.8734	.9643	.021	34	510	51,000	902
								2582

The values of ${}_tp_{x+1} = {}_tp_x\ (1-q_{x+t})$.

B

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19 Under the Unit Credit cost method, the accrued liability is defined as the present value of the accrued benefit. In this problem, you can work it very quickly, or you can spend a few extra minutes!

The key to the short solution is to consider the population and the assumptions. With no pre-retirement deaths or terminations, the only source of experience G/L is the assets.

$$eAAV_1 = (1+i)(AAV_0) + (\text{contrib}+i) - (BP+i)$$
$$= 1.07(19{,}000) + 8{,}000$$
$$= 28{,}330$$

$$\text{invest } G/L = AAV_1 - eAAV_1 = 26{,}000 - 28{,}330 = -2{,}330$$

(B)

The long way to work the problem is to calculate the total G/L as the difference between the expected and actual unfunded accrued liability. This requires calculation of the normal cost and accrued liability at 1/1/91 and 1/1/92:

	1-1-91 valn	1-1-92 valn
Age x	59	60
Past svc	4	5
Accd Benefit	4,800	6,000
$\ddot{a}_{65}^{(12)} D_{65}/D_x$	$8.74(1.07)^{-6}$	$8.74(1.07)^{-5}$
	$= 5.8238$	$= 6.2315$
Accd Liability	27,954	37,389
Normal cost	6,989 $= \frac{1}{4}(27{,}954)$	

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(19) continued

$G/L = eUAL_1 - UAL_1$

$eUAL_1 = (1+i)(NC_0 + UAL_0) - (\text{contrib} + i)$

$= 1.07\,(6{,}989 + 27{,}954 - 19{,}000) - 8{,}000$

$= 9{,}059$

$UAL_1 = 37{,}389 - 26{,}000$

$= 11{,}389$

$\Delta = G/L = 2{,}330$

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20 Under the Projected Unit Credit cost method, the normal cost and accrued benefit are calculated similar to the Unit credit method. The calculations use a Funding accrued benefit, (F.A.B.), that has the salary scale applied to retirement age.

This is the first problem ever asked with PUC, and a salary scale, and retirement decrements. You must calculate a separate FAB for each of the assumed retirement ages:

1-1-92 Age 35
Hire Age 33
Past service 2 years

1991 pay ⇒ Age 34 = 25,000
Age 64 = $25{,}000\,(1.06)^{30}$
= 143,587

FAE_3 at 64 = $143{,}587\,(\ddot{a}_{\overline{3}|.06}/3)$
= 135,613

FAE_3 at 61 = $135{,}613\,(1.06)^{-3}$
= 113,863

$FAB_{65} = 2\%(135{,}613)(2)$
$= 5{,}425$

$FAB_{63} = 2\%(113{,}863)(2)(1-.03(3))$
$= 4{,}145$

Key to use early retirement reduction

The accrued liability can be expressed with the usual formula for PV of ancillary benefits:

$$AL = PV \text{ of retirement benefits} = \sum v^t\,{}_tp_x\,q^r_{x+t}\,FAB_{x+t}\,\ddot{a}^{(12)}_{x+t}$$
$$= v^{27}\,{}_{27}p_{35}\,(.15)\,4{,}145\,\ddot{a}^{(12)}_{62} + v^{30}\,{}_{30}p_{35}\,(1.0)\,5{,}425\,\ddot{a}^{(12)}_{65}$$
$$= (1.08)^{-27}(1.0)(.15)(4145)(8.77) + (1.08)^{-30}(.85)(1.0)(5425)(8.20)$$
$$= 4{,}440$$

Key point is that only 85% of the participant is left to retire at age 65! (B)