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SPRING 1991 EA-1B EXAM SOLUTIONS

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1. Attained age normal is an aggregate cost method. The Unfunded Accrued Liability in the first year is (by definition) equal to the Unfunded Accrued Liability under the Unit Credit cost method. Since the plan is first effective in 1991, you must do a Unit Credit valuation to determine the UAL, then calculate the AANC as $\frac{PVNC}{\text{avg } PVL/L}$

1-1-91 age 50 service 20 total projected service 35

The Unit Credit Accrued Liability is defined as equal to the present value of the accrued benefit:

Accrued Benefit $20(\$120) = 2400$

P.V. of AB $2400\left(\frac{N_{65}^{(12)}}{D_{50}}\right) = 2400\left(\frac{849}{320}\right) = 6367.50$

The $UAL = AL - AAV$ but assets are zero in first year
$= 6367.50 - 0 = 6367.50$

$PVNC = PVB - UAL - AAV = PVB - 6367.50$ under AAN

PVB = Present value of projected benefit

Proj Ben $= 30(\$120) = 3600$ due to service limit of 30 years

$PVB = 3600(849/320) = 9551$

$PVNC = 9551 - 6367 = 3184$

avg $PVL/L = \ddot{a}_{50:\overline{15|}} = \frac{N_{50} - N_{65}}{D_{50}}$ $\qquad N_{65}^{(12)} = N_{65} - \frac{11}{24}D_{65}$

$= \frac{3716 + \frac{11}{24}(320) - [849 + \frac{11}{24}(97)]}{320}$ $\qquad \therefore N_{65} = N_{65}^{(12)} + \frac{11}{24}D_{65}$

$= 9.2788$

(B) 1/1/91 $NC = 3184 / 9.2788 = 343$

The only real trick here is the 30 year cap on benefit accruals!

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2. There are two ways to work this problem. One approach requires knowledge of the interest rate, but it is the truly general approach that works for all types of G/L questions.

The experience G/L (non-investment) is the difference between the actual and expected accrued liability:

non-inv G/L = $eAL_1 - AL_1$

AL_1 = Ben ($\ddot{a}_{x+1}$) if alive and zero if dead

$eAL_1 = (1+i)(NC_0 + AL_0) - (\text{actual } BP + i)$ note - annuity due

$= (1+i)(\text{Ben})(\ddot{a}_x) - (1+i)(\text{Ben})$ for retirees, NC = ∅

$= (1+i)(\text{Ben})(a_x)$

You can use the relationship $v\,p_x\,\ddot{a}_{x+1} = a_x$ to solve for $(1+i)$ at both age 60 and age 70:

$$v\,p_{60}\,\ddot{a}_{61} = a_{60} \qquad v\,p_{70}\,\ddot{a}_{71} = a_{70}$$

$$(1+i) = \frac{p_{60}\,\ddot{a}_{61}}{a_{60}} = \frac{.98(9.30)}{8.52} = 1.06972 \qquad (1+i) = \frac{p_{70}\,\ddot{a}_{71}}{a_{70}} = \frac{.96(7.00)}{6.28} = 1.07006$$

Apparently the actual interest rate should be 7.0%. Now you can calculate the eAL at 1/1/91:

$eAL_1 = 1.07(54.000)(9.52-1) + 1.07(24,000)(7.28-1)$

$= 653,556$

$AL_1 = 54,000\,(9.30) = 502,200$

Δ = G/L = 151,356 gain due to mortality, since $_eAL_1$ exceeds the actual value

D

The alternate approach is to derive the G/L directly based on annuity values and probabilities, without knowing the interest rate. This is discussed on the next page

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(2) (continued)

If you are brave, you could reason out what the G/L should be for a retiree who dies or survives. For example, we know that these statements are true for monthly annuities paid at the beginning of each month

Retiree who survives causes loss = $q_x \ddot{a}^{(12)}_{x+1} + \frac{11}{24} q_x$
equals liability loss + EBP loss

Retiree who dies causes gain $= (1+i)\ddot{a}^{(12)}_x - (\text{actual BP}+I)$
where the amount of actual BP depends on the day of death

Logically, if you have annual benefit payments, you would expect the following results

Retiree who survives causes loss = $q_x \ddot{a}_{x+1}$

Retiree who dies causes gain $= (1+i)\ddot{a}_x - (1+i)$
$= (1+i)a_x = p_x \ddot{a}_{x+1}$

The above results can be derived easily from the formula for the non-inv G/L on the prior page. Now you can plug in the values you are given to calculate the G/L:

Smith = loss = $q_{60}\ddot{a}_{61}(54{,}000) = .02(9.30)(54{,}000) = 10{,}044$
Brown = gain = $p_{70}\ddot{a}_{71}\ (24{,}000) = .96\ (7.00)\ (24{,}000) = 161{,}280$

Net gain = 151,236

Note - the arithmetic difference is due to the fact that the q_x and $\ddot{a}_x$ factors only have 3 significant digits, and the final answer only has the same 3 significant digits of accuracy.

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3. The key to this question is that the $eUAL_1$ is defined based on items held constant they way they were in last year's valuation. The G/L calculation should use the same cost method, assumptions, and benefit level as last year's valuation.

This year's accrued liability is based on the new \$12.50 benefit level. To calculate the UAL_1, we need the accrued liability on the old \$12.00 benefit level. Since the benefit change is effective for both actives and inactives, we define UAL_1 for G/L purposes as

$$UAL_1 = \frac{12.00}{12.50}(890{,}000) - 500{,}000$$
$$= 854{,}400 - 500{,}000 = 354{,}400$$

$$\text{Total G/L} = eUAL_1 - UAL_1$$

simple interest is acceptable

$$eUAL_1 = (1+i)(NC_0 + UAL_0) - (C+I)$$
$$= 1.07(50{,}000 + (800{,}000 - 420{,}000)) - 1.035(80{,}000)$$
$$= 460{,}100 - 82{,}800 = 377{,}300$$

C

$$\text{Total G/L} = 377{,}300 - 354{,}400$$
$$= 22{,}900$$

This is a gain, since the expected UAL exceeds the actual UAL at 1/1/91.

NOTE: If you used compound interest to calculate the eUAL, your final answer would be 22,947.

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4. Once you set up the old and the new normal cost calculations, this problem is simply lots of arithmetic:

old NC

$$23{,}615 = \frac{.5(40{,}000)\, N_{65}^{(12)}/D_{62} - AAV}{\ddot{a}_{62:\overline{3}|}}$$

new NC

$$= \frac{.5(40{,}000)(1.05)^2\, N_{65}^{(12)}/D_{62} - AAV}{{}^{s}\ddot{a}_{62:\overline{3}|}}$$

For the new normal cost, the benefit is based on final pay at age 64. The denominator is an annuity which includes salary scale increases, so that the normal cost is funded as a level percentage of pay.

$$\begin{aligned} AAV &= 20{,}000\,(N_{65} - \tfrac{11}{24}D_{65})/D_{62} - 23{,}615\,(N_{62} - N_{65})/D_{62} \\ &= 20{,}000\,(2508 - \tfrac{11}{24}(279))/365 - 23{,}615\,(3514 - 2508)/365 \\ &= 20{,}000\,(6.5209) - 23{,}615\,(2.7562) \\ &= 65{,}331 \end{aligned}$$

For the new normal cost calculation, the only potential trick is calculating the annuity in the denominator:

$$\begin{aligned} {}^{s}\ddot{a}_{62:\overline{3}|} &= ({}^{s}N_{62} - {}^{s}N_{65})/{}^{s}D_{62} \\ &= ({}^{s}D_{62} + {}^{s}D_{63} + {}^{s}D_{64})/{}^{s}D_{62} \\ &= \left((1.05)^{62}D_{62} + (1.05)^{63}D_{63} + (1.05)^{64}D_{64}\right)/\left[(1.05^{62})D_{62}\right] \\ &= 1 + \frac{(1.05)D_{63}}{D_{62}} + \frac{(1.05)^2 D_{64}}{D_{62}} \\ &= 1 + \frac{1.05(335) + (1.05)^2(306)}{365} \\ &= 2.8880 \end{aligned}$$

$$\text{new NC} = \frac{20{,}000\,(1.05)^2(6.5209) - 65{,}331}{2.8880} = 27{,}166$$

(B)

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5. The key to this problem is the ability to express the optional forms of benefit payment in a formula. Since the options are actuarially equivalent to the straight life annuity, you can solve for various missing items

1-1-91
Partic age 65
Spouse age 62

Normal form PVB $= 2500\,\ddot{a}_{65}$

Option A PVB =

$A\,(\ddot{a}_{65} + .5(\ddot{a}_{62} - \ddot{a}_{65:62}))$

Option B PVB =

$2376\,(\ddot{a}_{62:62} + .5(\ddot{a}_{62} - \ddot{a}_{65:62}) + .5(\ddot{a}_{65} - \ddot{a}_{65:62})) = 2376\,(.5)(\ddot{a}_{62} + \ddot{a}_{65})$

The definition of the PVB for optional form B is the hardest part of the problem. The original formula expresses the value as an annuity payable as long as both the participant and spouse are alive, plus a reversionary annuity to the spouse after the participant's death, and to the participant after the spouse's death. The final simplified form clearly pays 2376 when both are alive, and ½ to any sole survivor.

Since the optional forms are actuarially equivalent, the present values must all be equal.

$2500\,\ddot{a}_{65} = 26{,}000 \quad \therefore \ddot{a}_{65} = 10.40$

Option B: $26{,}000 = 1188\,(\ddot{a}_{62} + \ddot{a}_{65})$

$\ddot{a}_{62} = (26{,}000/1188) - \ddot{a}_{65}$

$= 11.4855$

Option A: $26{,}000 = A\,(\ddot{a}_{65} + .5[\ddot{a}_{62} - \ddot{a}_{65:62}])$

$A = 26{,}000/(10.40 + .5[11.4855 - 9.42])$

$= 2{,}274$

D

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6\. There are two different ways of working this problem, which is quite similar to problem #2 on this exam. Based on the discussion in that problem, we show that, for an annual life annuity due,

Loss due to survival of retiree = Benefit $(q_x \ddot{a}_{x+1})$
Gain due to death of retiree = Benefit $(p_x \ddot{a}_{x+1})$

Based on the information you are given, you must derive the value of $\ddot{a}_{x+1}$. The key is that e_x is basically a_x without any interest (or interest = 0%). The key formula relates the values of two successive annuities:

$$v p_x \ddot{a}_{x+1} = a_x \qquad \Rightarrow \qquad p_x(1+e_{x+1}) = e_x$$

$$\ddot{a}_{x+1} = \frac{(1+i)a_x}{p_x} \qquad\qquad p_x = \frac{e_x}{1+e_{x+1}}$$

$$\therefore \quad p_{70} = \frac{e_{70}}{1+e_{71}} = \frac{13.80}{14.25} = .9684$$

$$\therefore \quad \ddot{a}_{71} = (1.07)a_{70} / p_{70} = 1.07(6.326)/.9684 = 6.9895$$

Loss due to survival $= 10{,}000(q_{70})\ddot{a}_{71} = 10{,}000(6.9895)(.0316)$
$= 2{,}207$

(B)

The alternate approach is to use the general formula for non-investment G/L:

non-inv G/L $= eAL_1 - AL_1$

$AL_1 = 10{,}000\, \ddot{a}_{71} = 69{,}895$ based on $\ddot{a}_{71}$ above

$eAL_1 = (1+i)(NC_0 + AL_0) - (\text{actual } BP+i)$

$= 1.07(\emptyset + 10{,}000(7.326)) - 1.07(10{,}000)$

$= 1.07(10{,}000)(6.326)$

$= 67{,}688$

$\Delta = 2{,}207$

The second approach is slightly longer - you have to derive $\ddot{a}_{71}$ in both!

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7. The real key to this problem is knowledge of the relationship between $A_x^{(m)}$ and $\ddot{a}_x^{(m)}$. This really tests material covered on the 1A exam.

$$A_x = 1 - d\ddot{a}_x \qquad A_x^{(m)} = 1 - d^{(m)}\ddot{a}_x^{(m)}$$

$$\left[1 - \frac{d^{(m)}}{m}\right]^{-m} = 1+i \quad \Rightarrow \quad 1 - \frac{d^{(m)}}{m} = (1+i)^{-1/m}$$

$$d^{(m)} = m\left[1 - (1+i)^{-\frac{1}{m}}\right]$$

$$5000\,A_{65}^{(12)} = 2100 \quad \Rightarrow \quad A_{65}^{(12)} = 21/50$$

$$\ddot{a}_{65}^{(12)} = \frac{1 - A_{65}^{(12)}}{d^{(12)}} \qquad d^{(12)} = 12\left[1 - (1.07)^{-\frac{1}{12}}\right] = 6.747\%$$

$$= \frac{29/50}{.06747} = 8.5966$$

Now that you have the value of $\ddot{a}_{65}^{(12)}$, you can calculate the "funding accrued benefit" under the projected Unit Credit cost method. You are told to use a service prorate method to assign these flat benefits to years of service.

Year	Age	Service	Pension Ben	Death Ben
1-1-91	60	25	(25/30)(6000)	(25/30)(5000)
1-1-92	61	26	(26/30)(6000)	(26/30)(5000)
			Δ (1/30)(6000)	(1/30)(5000)

Normal cost = PV of (Δ Funding Accrued Benefit)

$$= v^5 (200)\,\ddot{a}_{65}^{(12)} + v^5 (166.67)\,A_{65}^{(12)}$$

$$= (1.07)^{-5}\left[200(8.5966) + 166.67(21/50)\right]$$

$$= 1276$$

B

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8. This problem is relatively easy as long as you don't try to back into $\ddot{a}_{65}^{(12)}$. You should express the normal cost via formula each year and take the ratio.

Under the Projected Unit Credit method, the normal cost is the present value of the change in the "Funding accrued benefit". In a plan with a uniform rate of benefit accrual for all years of service, the change in the FAB is constant:

$$\text{1-1-90 NC} = 1\% \,(FAE_3 \text{ at } 64)\, N_{65}^{(12)} / D_{55}$$

$$\begin{array}{l}\text{True expected}\\ \text{1-1-91 NC}\end{array} = p_{55}(1\%)(FAE_3 \text{ at } 64)\, N_{65}^{(12)} / D_{56} \quad (p_x \text{ survivors})$$

Based on salary increase expected to occur, the FAE_3 at age 64 would be the same as last year

$$\begin{array}{l}\text{Actual}\\ \text{1-1-91 NC}\end{array} = 1\% \left(\frac{1.08}{1.05}\right)(FAE_3 \text{ at } 64)\, N_{65}^{(12)} / D_{56}$$

$$\text{Ratio } \frac{\text{1-1-91 NC}}{\text{1-1-90 NC}} = \frac{1.08}{1.05}\left(\frac{D_{55}}{D_{56}}\right) = \frac{1.08\, v^{55} l_{55}}{1.05\, v^{56} l_{56}}$$

$$= \frac{1.08\,(1+i)}{1.05\,(l_{56}/l_{55})} = \frac{1.08\,(1.07)}{1.05\,(1-.009)} = 1.1106$$

(D)

In general, if all assumptions are met and you have a uniform rate of benefit accrual, the normal cost will increase at the rate of $(1+i)$ each year. For an individual who survives, the normal cost increase is $(1+i)/p_x$. Here you also had a salary scale increase on top of the $\frac{1+i}{p_x}$ increase.

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9. This problem has more to do with EA-1A material than with EA-1B! You should use the general expression for ancillary benefits to express the PV of the death benefit:

$$\sum_{t=0}^{4} v^{t+1} {}_tp_x \, q_{x+t}^{(d)} (5000)(\text{service at } x+t+1) = \sum_{t=0}^{4} v^{t+1} \frac{d_{x+t}}{l_x} [5000][21+t]$$

$$= \sum_{t=0}^{4} \frac{v^{x+t+1} d_{x+t}}{v^x l_x} [5000][21+t] = \frac{5000}{D_x} \sum_{t=0}^{4} C_{x+t}(21+t)$$

This present value represents an increasing term insurance benefit for a period of five years. The participant's age x is 60 at 1/1/91. One potential trick is the definition of years of service to count in the year of death. The participant has 20 years of service at 1/1/91, but the death benefit allows for 21 years upon death at any time during 1991.

To value this death benefit, the quickest approach is to construct the C_x values:

$$C_x = v^{x+1} d_x = v^{x+1}(l_x - l_{x+1}) = v^{x+1} l_x - v^{x+1} l_{x+1}$$
$$= v D_x - D_{x+1}$$

Age	C_x
60	196.65
61	198.13
62	200.70
63	201.76
64	203.50

$$PV \text{ death ben} = \frac{5000}{14863}\big(21(196.65) + 22(198.13) + 23(200.70) + 24(201.76) + 25(203.50)\big)$$
$$= 7749$$

C

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10. Under Projected Unit Credit, the accrued liability is defined as the present value of the "Funding Accrued Benefit" (FAB). The FAB is defined equal to the projected benefit at assumed retirement age, multiplied by a fraction. The fraction is based on the ratio of past service to total service, but the years are weighted by the rate of benefit accrual attributable to each year.

In general, the FAB can be calculated by applying the benefit formula to past service, and using projected earnings to assumed retirement age. This gives the same result as the more complicated procedure described above.

1-1-91 age = 40 Past service = 2 years

<u>Assumed retirement at age 65</u>

$FAB = (2\text{ years})(2\%)(FAE_3 \text{ at age } 64)$

$= 2(.02)(21{,}000)(1.05)^{24}(\ddot{a}_{\overline{3}|.05}/3)$

$= 2582$

$AL = 2582\, \ddot{a}_{65}^{(12)}\, D_{65}/D_{40}$ but no pre-ret decrements

$= 2582\,(8.75)(1.07)^{-25}$

$= 4163$

<u>Assumed retirement at age 62</u>

You must be careful to project earnings to age 61, and to apply the age reduction based on assumed retirement age

$FAB = 2(.02)(21{,}000)(1.05)^{21}(\ddot{a}_{\overline{3}|.05}/3)(1-3[1/15])$

$= 1784$

$AL = 1784\, \ddot{a}_{62}^{(12)}\, D_{62}/D_{40}$

$= 1784\,(9.40)(1.07)^{-22}$

$= 3786$

(A) ΔAL = decrease of 377

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11. This problem requires that you know how to apply Entry Age Normal "on an aggregate basis". The typical calculation of the Entry Age normal cost on an individual basis is $PVB_{EA} / \ddot{a}_{EA:\overline{RA-EA}|}$. With many participants, the total EANC $= \sum \left(\frac{PVB_{EA}}{\ddot{a}_{EA:\overline{RA-EA}|}} \right)$.

For Entry Age Normal "on an aggregate basis", you move the summation symbol into the numerator and denominator:

$$\text{Aggregate EANC} = \frac{\sum PVB_{EA}}{\underbrace{\frac{\sum \ddot{a}_{EA:\overline{RA-EA}|}}{\sum 1}}_{\text{avg annuity}}}$$

	AGE 40	AGE 60	TOTAL
(1) Number of participants	2	2	
(2) Past service at 1-1-91	10	10	
(3) Entry age	30	50	
(4) Total service at 65	35	15	
(5) Projected benefit at 65	4200	1800	
(6) PV factor $N_{65}^{(12)} / D_{EA}$	873/1336 = .6534	873/329 = 2.6535	
(7) PVB at entry age (1)x(5)x(6)	5489	9553	15042
(8) $\ddot{a}_{EA:\overline{RA-EA}|}$	(18946-919)/1336 = 13.4933	(3974-919)/329 = 9.2857	
(9) Individual EANC (7)÷(8)	407	1029	1436
	$= \frac{2744}{13.49} + \frac{2744}{13.49}$	$= \frac{4776}{9.29} + \frac{4776}{9.29}$	

For all four employees, the average $\ddot{a}_{EA:\overline{RA-EA}|}$ is calculated as $\frac{2(13.4933) + 2(9.2857)}{4} = 11.3895$.

The total "Aggregate EANC" is $\frac{15{,}042}{11.3895} = 1321$

Δ EANC $= 1436 - 1321$
$= 115$

B

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12. This problem requires you to determine the effect of both a plan change and a pay raise on the 1-1-91 normal cost. The safe way to work this problem is to set up three columns for the 1-1-90 valuation, 1-1-91 expected results, and final 1-1-91 valuation. You should show that the 1-1-91 expected normal cost is a constant percentage of pay (versus 1-1-90).

There will be numerous holes in these three columns that are filled in based on the results of various calculations:

		1-1-90	expected 1-1-91	Actual 1-1-91
	PVB	700,000	749,000	
	AAV	141,250	201,138	
	UAL	275,000	271,000	
Balancing	PVNC	283,750	276,863	
	PVE	5,675,000	5,537,250	
	Earn	500,000		
	PVE/E	11.35		
	NC	25,000		
	% Pay	5.00	5.00	

1-1-90 PVNC = 11.35(25,000) = 283,750 , AAV = 700,000 - 275000 - 283,750 = 141,250

1-1-91 eUAL = 1.07(25,000 + 275,000) - 50,000 = 271,000
ePVB = 1.07 (700,000) = 749,000
eAAV = 1.07 (141,250) + 50,000 = 201,138
PVNC = PVB - UAL - AAV = 276,863
ePVE = 1.07 (5,675,000 - 500,000) = 5,537,250
% Pay = 276,863 / 5,537,250 = 5.00%

Must adjust UAL for plan change, use Δ EAN AL:
40% Pay EAN AL = 550,000
30% Pay EAN AL = 550,000 (3/4) = 412,500
Δ EAN AL = 137,500

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(12) (Continued)

1-1-91 Actual

$PVB = (4/3)(1.10/1.05)\,749{,}000 = 1{,}046{,}222$

$AAV = \qquad = 250{,}000$ given

$UAL = 271{,}000 + 137{,}500 = 408{,}500$

$PVNC = PVB - AAV - UAL = 387{,}722$

$PVE = (1.10/1.05)\,5{,}537{,}250 = 5{,}800{,}929$

$Earn = 1.10\,(500{,}000) = 550{,}000$

$PVE/E = 5{,}800{,}929/500{,}000 = 10.5471$

$NC = 387{,}722/10.5471 = 36{,}761$

E

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13. The only trick to this problem is realizing that Green is retired at 1-1-91. When you calculate the average annuity for funding purposes, you should only include Smith and Brown in the average.

	Smith	Brown	Green	Total
Status	Active	Active	Retired	
1-1-91 Age	45	55	65	
Projected Benefit	20,000	20,000	20,000	
PV factor	$8.74(1.07)^{-20}$	$8.74(1.07)^{-10}$	8.74	
PV Benefits	45,172	88,859	174,800	308,831
$\ddot{a}_{x:\overline{65-x}\rceil}$	$\ddot{a}_{\overline{20}\rceil .07}$ = 11.3356	$\ddot{a}_{\overline{10}\rceil .07}$ = 7.5152	N/A	18.8508

PVNC = PVB − AAV − UAL
= 308,831 − 208,800 − 12,000
= 88,031

Average
$\ddot{a}_{x:\overline{65-x}\rceil}$ = 9.4254 (based only on active ees)

NC = 88,031 / 9.4254
= 9,340

1/1/91 contrib = $9{,}340 + \frac{12{,}000}{\ddot{a}_{\overline{10}\rceil .07}}$ = 10,937

C

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14. Under the aggregate method, the PVNC is defined equal to PVB - AAV. With employee contributions, the PVB includes refunds of employee contributions, and the AAV includes the PV of future employee contributions.

You should do two valuations, one with employee contributions and one without them:

	With EEC	No EEC	
PV Retirement Bens	10,000,000	10,000,000	
PV future refunds	850,000		
Total PVB	10,850,000	10,000,000	
PV Future EEC	2,000,000	-0-	
Actuarial asset	4,800,000	4,800,000 - 1,200,000 refunded	
Total AAV	6,800,000	3,600,000	
PVNC = PVB - AAV	4,050,000	6,400,000	
PVE/E = $\frac{50,600}{3,500}$	14.4571	14.4571	
Normal Cost	280,138	442,688	Δ = 162,549

(E)

The really tricky part of this question is the reduction in the AAV due to refunding the accumulated contributions with interest to the participants.

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15. Under the FIL method, the PVNC is equal to PVB-UAL-AAV. When the plan benefits are changed, the UAL is adjusted by the effect of the plan change on the Entry Age Normal accrued liability. The main trick to this question is that you can't simply pro-rate both the PVB and the UAL - you will get the wrong answer!

1-1-91 Age=55 Past service=5 years Entry age=50 Total svc=15 years

\$10 $EANC = PVB_{EA} / \ddot{a}_{EA:\overline{RA-EA}|}$

$$= \frac{(\$120)(15\text{ years})\ \ddot{a}_{65}^{(12)}\ (D_{65}/D_{50})}{(N_{50}-N_{65})/D_{50}} = \frac{1800(8.74)(28{,}570)}{1{,}135{,}407-262{,}659}$$

$$= 515$$

\$10 $EAN\ AL = PVFB - PVNC$

$$= 1800\,(\ddot{a}_{65}^{(12)})\,D_{65}/D_{55} - 515\ \ddot{a}_{55:\overline{10}|}$$

$$= 1800(8.74)(28{,}570/64{,}742) - 515(727{,}747-262{,}659)/64{,}74$$

$$= 6942 - 515(7.1837) = 3243$$

$\Delta EANAL = (15/10)(3243) - 3243 = 5/10(3243) = 1621$

\$15 $PVB = 6942(15/10) = 10{,}413$

\$15 $UAL = 1800 + 1621 = 3{,}421$

$AAV = \qquad = 1{,}000$

\$15 $PVNC = PVB-UAL-AAV = 5{,}992$

\$15 $NC = 5{,}992 / \ddot{a}_{55:\overline{10}|} = 5{,}992/7.1837 = 834$

\$10 $PVNC = 6{,}942 - 1{,}800 - 1000 = 4{,}142$

\$10 $NC = 4{,}142 / \ddot{a}_{55:\overline{10}|} = 577$

$\Delta NC = 834 - 577 = 257$

B

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16. Under the Aggregate method, the PVNC is defined as PVB - AAV. With a % of pay benefit, you would usually calculate the normal cost to be a level % of pay. However, this problem explicitly states that you should calculate a level dollar normal cost.

The Individual Aggregate method requires you to calculate each employee's PVNC as PVB-AAV. The AAV has to be allocated to each employee based on some rule. In this problem, there are no assets, so you don't have an allocation rule.

		<u>Smith</u>	<u>Brown</u>	<u>Total</u>	
(1)	1-1-91 age	60	30		
(2)	Past service	25	0		
(3)	Total service	30	35		
(4)	Projected pay at 64	90,000	12,000		
(5)	Projected benefit	54,000	8,400		
(6)	PV factor	$v^5 \ddot{a}_{65}^{(12)}$ = 6.36	$v^{35} \ddot{a}_{65}^{(12)}$ = .6320		
(7)	PVFB	343,443	5,309	348,752	
(8)	$\ddot{a}_{x:\overline{65-x}\rceil}$	$\ddot{a}_{\overline{5}\rceil .08}$ = 4.3121	$\ddot{a}_{\overline{35}\rceil .08}$ = 12.5869	16.8991	avg = 8.4495
(9)	Allocated AAV	0	0	0	
(10)	Indiv AGG NC = [(7)-(9)]/(8)	79,646	422	80,068	

$$\text{AGG NC} = \frac{348{,}752 - 0}{8.4495} = 41{,}275$$

Shortfall = 38,793

(B)

The main reason for the shortfall is that, under the Individual Aggregate method, the largest liability is amortized over the shortest period.

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17. This problem is similar to several others on this exam which require a great deal of analysis to get the right answer. This problem can be worked more quickly because you only have one participant.

If all assumptions were met during 1990, your normal cost would remain level at 1296. You simply have to adjust the normal cost for the effect of the investment gain of 500:

1-1-91 Age = 46

Expected 1-1-91 NC = 1296 = PVNC / $\ddot{a}_{\overline{19}|.08}$

Actual 1-1-91 NC = (PVNC − 500) / $\ddot{a}_{\overline{19}|.08}$

(A) = 1296 − 500 / $\ddot{a}_{\overline{19}|.08}$ = 1248

The long way to work this problem is to set up three collumns for the 1990 valuation, expected results at 1-1-91, and the actual results at 1-1-91:

	1-1-90	Expected 1-1-91	Actual 1-1-91
PVB	29,560	31,925	31,925
UAL	12,818	12,243	12,243
AAV	3,000	6,240	6,740
PVNC	13,742	13,442	12,942
PVL	$\ddot{a}_{\overline{20}|.08}$ = 10.6036	$\ddot{a}_{\overline{19}|.08}$ = 10.3719	10.3719
NC	1,296	1,296	1,248

1-1-90 PVNC = 1296(10.6036) = 13,742 PVB = UAL + AAV + PVNC = 29,560

expected 1-1-91 ePVB = 1.08 (29,560) = 31,925

eUAL = 1.08 (1,296 + 12,818) − 3,000 = 12,243

eAAV = 1.08 (3,000) + 3,000 = 6,240

ePVNC = ePVB − eUAL − eAAV = 13,442

e NC = 13,442 / $\ddot{a}_{\overline{19}|.08}$ = 1,296

actual 1-1-91 AAV = 500 gain + 6,240 = 6,740

PVNC = 31,925 − 12,243 − 6,740 = 12,942

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18. Under the Individual Level Premium cost method, the Normal cost funds any change in benefit over future service. In general, this includes a change in projected benefit due to pay increases in excess of those assumed.

The point of this problem is that the ILP normal cost is first calculated at 1-1-82, based on the participant's age at 1-1-82.

1-1-82 Age = 32 Past service = 1 Total service = 34

$$ILP\ NC = \frac{PVB \text{ at age } 32}{\ddot{a}_{32:\overline{33}|}} = \frac{34(\$25)(12)\,\ddot{a}_{65}^{(12)}\,(D_{65}/D_{32})}{\ddot{a}_{32:\overline{33}|}}$$

$$= \frac{10{,}200\,\ddot{a}_{65}^{(12)}\,D_{65}}{N_{32}-N_{65}} = \frac{10200\,(8.5)(200)}{24{,}000-1792} = 781$$

1-1-91 Age = 41

$$PVNC = (ILP\,NC)\,\ddot{a}_{41:\overline{24}|}$$

$$= 781\left(\frac{N_{41}-N_{65}}{D_{41}}\right) = 781\left(\frac{13050-1792}{900}\right)$$

$$= 9767$$

E

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19. There are two tricky aspects to this problem. The first one is the complications in setting up the present value of the early retirement benefits with plan provisions and assumptions that vary based on years of service.

The second confusing area is the D_x values that you are given. You are told that there is no preretirement death or termination assumption. If you try to calculate $vp_x = D_{x+1}/D_x$, you discover that p_x is about .98, which seems to imply that the D_x's include mortality. The only reason you are given the D_x's and $N_x^{(12)}$ values is so you can calculate $\ddot{a}_x^{(12)}$ values!

Here is a general expression for the present value of benefits for a participant at age 62:

$$PVB = \sum_{t=0}^{3} v^t \, {}_tp_{62}^{(T)} \, q_{62+t}^{(r)} \, ERB_{62+t} \, \ddot{a}_{62+t}^{(12)}$$

t	Age ($62+t$)	Service	v^t	$q_{62+t}^{(r)}$	${}_tp_{62}^{(T)}$	ERB_{62+t}	$\ddot{a}_{62+t}^{(12)}$
0	62	28	$(1.07)^0$	.10	1.000	\$180(28)(1−3(6%))	34,796/3704
1	63	29	$(1.07)^{-1}$	.10	.900	\$180(29)(1−2(6%))	31230/3403
2	64	30	$(1.07)^{-2}$	.40	.810	\$180(30)	27956/3121
3	65	31	$(1.07)^{-3}$	1.00	.486	\$180(30)	24956/2857

One trick is that benefit service is limited to 30 years!

$$\begin{aligned} PVB &= 1(.1)(1)(5040)(.82)(9.3942) + .9346(.1)(.9)(5220)(.88)(9.1772) \\ &\quad + .8734(.4)(.81)(5400)(8.9574) + .8163(1)(.486)(5400)(8.7350) \\ &= 3{,}882 + 3{,}546 + 13{,}688 + 18{,}713 \\ &= 39{,}830 \end{aligned}$$

B

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20. Under the Individual Level Premium cost method, the Normal Cost funds any change in benefit over future service. This includes changes in benefit due to salary increases in excess of those assumed.

		1-1-90	1-1-91	
(1)	Age	42	43	
(2)	Pay	43,500	46,000	
(3)	Projected Pay at 64	$43{,}500(1.04)^{22}$ = 103,091	$46{,}000(1.04)^{21}$ = 104,823	
(4)	FAE$_3$ at 64 = (3) $\ddot{a}_{\overline{3}|4\%}$ / 3	99,177	100,843	
(5)	Projected Benefit .75 (4)	74,383	75,633	$\Delta = 1250$

The 1991 ILP normal cost will equal the 1990 ILP normal cost plus the additional layer of normal cost to fund the increase in projected benefit over future service. You would normally determine the layers as a level % of pay, but this problem tells you to calculate the normal cost as a level dollar amount.

$$\Delta NC = \frac{(\Delta \text{Proj Ben})\, N_{65}^{(12)} / D_x}{\ddot{a}_{x:\overline{65-x}|}}$$

$$= \frac{1250\,(8.5)\,(1.08)^{-22}}{\ddot{a}_{\overline{22}|.08}}$$

$$= \frac{1250\,(8.5)}{\ddot{s}_{\overline{22}|.08}} = 177$$

(B)