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SPRING 1990 EA-1B EXAM SOLUTIONS

Revision History:
03/08/99 Corrected problem 5 - allow for mortality decrement at beginning of year

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1. Under any aggregate cost method, you must reflect both the PV of future employee contributions and the value of refunds of employee contributions in the balance sheet:

	AAV	PVB – actives
	PVEEC	PVB – nonactives
	UAL	PV future refunds
balancing	PVNC	
	Σ PVFB	Σ PVFB

Since employees contribute 1.5% of pay, you can approximate the PV of future employee contributions as .015(8,000,000) = 120,000. I say this is an approximation because the expressions for PVE and PV EEC are not really comparable; assuming contributions are paid EOY

$$PVE = \text{Earnings}\,(1 + v\,p_x(1+s) + v^2\,{}_2p_x(1+s)^2 + \ldots)$$

$$PV\ EEC = \text{Earnings}\,(.015)\,(v + v^2 p_x(1+s) + {}_2p_x\, v^3(1+s)^2 + \ldots)$$

400,000	AAV	1,500,000	PVB retirements
,015(8,000,000) = 120,000	PVEEC	30,000	PV refunds
–0–	UAL		
1,010,000	PVNC		
1,530,000		1,530,000	

Employer

$$NC = \frac{1{,}010{,}000}{8{,}000{,}000/1{,}000{,}000} = 126{,}250$$

(A)

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2. You must set up the balance sheet at 1-1-91 to determine the assets. I recommend setting up the true expected balance sheet first which should produce the same NC as % of pay at 1-1-90.* Then make adjustments to reflect the final true cost calculation:
(1) Plan benefits change at 1-1-91
(2) Given normal cost is based on new plan benefits
(3) 8% salary increase instead of 5% expected
(4) Unknown asset return during 1990

<u>1-1-90 Balance Sheet</u>

PVNC = PVB − AAV = 800,000 − 300,000 = 500,000
NC% = PVNC/PVE = 500,000/11,250,000 = 4.44%
NC = 4.44% (900,000) = 40,000

<u>1-1-91 Expected Balance Sheet</u>

ePVB = 1.07 (800,000) 856,000
eAAV = 1.07 (300,000 + 40,000) = 363,800
ePVNC = 856,000 − 363,800 = 492,200
ePVE₁ = 1.07 (PVE₀ − E₀) = 1.07(11,250,000 − 900,000) = 11,074,500
NC% = PVNC/PVE = 492,200/11,074,500 = 4.44%
eEarn = 1.05 (900,000) = 945,000
eNC = 4.44% (945,000) = 42,000 (also equals 1.05(40,000))

<u>1-1-91 Actual Balance Sheet</u>

PVB = 856,000 (50%/40%)(1.08/1.05)
= 1,100,571
Earn = 945,000 (1.08/1.05) = 972,000
PVE = 11,074,500 (1.08/1.05) = 11,390,914

$$NC = 60{,}000 = \frac{PVNC}{PVE/E} = \frac{PVB - AAV}{11.719}$$

∴ AAV = PVB − 60,000 (11.719)
= 1,100,571 − 703,143
= 397,428

E

Within the implied range of 395,000 to 405,000

* Under the Aggregate method, this requires 1/1 contribution = normal cost.

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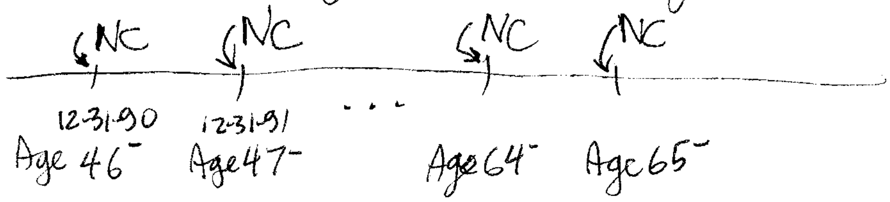
3. With EOY valuation, must be careful to use correct # years in calculation of the normal cost. Under the Aggregate method, PVNC = PVB − AAV. In this problem you have to calculate the PVB for active and retired employees.

	Smith	Brown	
1-1-91 Age	65	46	
Annual benefit	12,000	15,000	= .5(30,000)
Future service	0	19	
$(1.07)^{-FS}$	1	.2765	
PVB at 1-1-91	12,000(8.74)	15,000(8.74)(.2765)	
	104,880	36,250	

Total PVNC = 104,880 + 36,250 − 94,650
= 46,480

Now look at time line diagram for number of years to calculate annuity for Brown's future service years:



At 12-31-90, the present value of this series of payments is NC($\ddot{a}_{\overline{20|}.07}$)

$\therefore$ NC = $\dfrac{46,480}{\ddot{a}_{\overline{20|}.07}}$ = 4100

D

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4. Key to this question is how to determine the entry age normal cost. With a pay-related formula, the normal cost should be a level percentage of pay. The only trick is to calculate the old Normal cost on the 1% of pay formula, and adjust it for the 1.25% of pay formula.

$UAL = AL - AAV$

$AL = PVB - PVNC = 4100 - PVNC$

$= UAL + AAV = 800 + 1000$

$\therefore PVNC = 4100 - 1800 = 2300$

$$NC\ \% = \frac{PVNC}{PVE} = \frac{2300}{46000} = 5\%$$

Since the $EANC\% = \dfrac{PVB_{EA}}{PV\,Sal_{EA}}$, a change in the benefit formula from 1% to 1.25% should give a Normal cost that is $5\%(1.25/1.00) = 6.25\%$ of pay.

New $EANC = .0625(4000) = 250$

E

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5. There are two points to this question. The first is that the normal cost consists of two components, a portion for retirement benefits and a portion for vesting benefits. The second (major) point is that

1) under the Unit Credit cost method
2) if a participant is 100% vested
3) valuation results are independent of withdrawal rates

This means that we can do a very simple calculation of the normal cost by setting the withdrawal rates to zero, which means we only have to calculate a retirement component of the normal cost:

1-1-90 Age 63
Service 5
Accd Benefit 600
Δ Accd Benefit 120

Under unit credit, the normal cost is the PV of one year's benefit accrual:

$$NC = 120\,\ddot{a}_{65}^{(12)}\,\frac{D_{65}^{T}}{D_{63}^{T}} = 120\,\ddot{a}_{65}^{(12)}\,v^{2}\,{}_{2}p_{63}^{T}$$

$$= 120(8.736)(1.07)^{-2}(1-.019)(1-.021)$$

$$= 879$$

C

See the next page for alternate multiple decrement calculation of the normal cost

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(5) The hard way to work the problem is reflecting both the mortality and termination decrements. The complication is that the mortality occurs at the beginning of the year, and terminations other than death are at the end of the year!

NC = PV of benefit accrual

$$= 120\ddot{a}_{65}^{(12)} \frac{D_{65}^{(T)}}{D_{63}^{(T)}} + 120v(1-q_{63}^{(d)})q_{63}^{(w)} \frac{D_{65}}{D_{64}} \ddot{a}_{65}^{(12)} + 120v^2(1-q_{63}^{(d)})(1-q_{63}^{(w)})(1-q_{64}^{(d)})q_{64}^{(w)} \ddot{a}_{65}^{(12)}$$

Retire @ 65 withdraw EOY age 63 withdraw EOY age 64

$$= 120\ddot{a}_{65}^{(12)}\left[v^2(1-q_{63}^{(d)})(1-q_{63}^{(w)})(1-q_{64}^{(d)})(1-q_{64}^{(w)}) + v(1-q_{63}^{(d)})q_{63}^{(w)}(1-q_{64}^{(d)})v + v^2(1-q_{63}^{(d)})(1-q_{63}^{(w)})(1-q_{64}^{(d)})q_{64}^{(w)}\right]$$

$$= 120v^2\ddot{a}_{65}^{(12)}[(1-.019)(1-.05)(1-.021)(1-.06) + (1-.019).05(1-.021) + (1-.019)(1-.05)(1-.021).06]$$

$$= 915.64\,[.85764 + .04802 + .05474]$$

$$= 879.38$$

(C)

If you start with the expression based on q_x terms only, you can simplify it to that shown on the prior page:

$$(1-q_{63}^{(d)})(1-q_{63}^{(w)})(1-q_{64}^{(d)})(1-q_{64}^{(w)}) + (1-q_{63}^{(d)})q_{63}^{(w)}(1-q_{64}^{(d)}) + (1-q_{63}^{(d)})(1-q_{63}^{(w)})(1-q_{64}^{(d)})q_{64}^{(w)}$$

$$= (1-q_{63}^{(d)})(1-q_{64}^{(d)})\left[(1-q_{63}^{(w)})(1-q_{64}^{(w)}) + q_{63}^{(w)} + (1-q_{63}^{(w)})q_{64}^{(w)}\right]$$

$$= (1-q_{63}^{(d)})(1-q_{64}^{(d)})\left[(1-q_{63}^{(w)}) + q_{63}^{(w)}\right]$$

$$= (1-q_{63}^{(d)})(1-q_{64}^{(d)}) \qquad \text{Q.E.D.}$$

In the original version of this solution, I had not noticed (or used) the fact that mortality decrements occur at the beginning of the year – but I was still able to simplify the present value expression to the identical result shown above.

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6. The quickest way to calculate the accrued liability is with the retrospective approach:

$$AL_x = \sum_{t=EA}^{x-1} NC_t \frac{D_t}{D_x} = NC_{EA} \frac{(N_{EA} - N_x)}{D_x}$$
$$= NC_{EA}\, \ddot{s}_{EA:\overline{x-EA}|}$$

With no pre-retirement decrements, this becomes an annuity certain. The change in the accrued liability is the difference in the normal cost before and after the amendment multiplied by the annuity. With a dollar per month plan, the revised normal cost will be (\$12/\$10) times the old normal cost

	Smith	Brown
Hire Age	25	35
1-1-90 Age	30	40
Service at 65	40	30
\$120 projected benefit	4800	3600
\$120 normal cost	202.224	320.544
\$144 projected benefit	5760	4320
\$144 normal cost	242.67	384.65
Δ normal cost	40.44	64.11
$\Delta AL = \ddot{s}_{\overline{5}|.07}(\Delta NC)$	248.87	394.48

(this is 20% of the prior NC)

Σ = 643 which is in the implied range of 625-725

(E)

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7. Under the FIL method, the Normal cost equals the PVNC divided by the average temporary annuity. Since this problem tells us the normal cost is a level dollar amount, the temporary annuity does not include a salary scale. With no pre-retirement deaths or terminations, the temporary annuity is an annuity certain.

$PVNC = PVB - UAL - AAV$
$= PVB - 10{,}000 - 91{,}200$

	Smith	Brown	Green	
1-1-90 Age	70	35	45	
Annual benefit	12,000	60,000	36,000	
PV factor	$\ddot{a}'^{(12)}_{70}$	$v^{30}\ddot{a}^{(12)}_{65}$	$v^{20}\ddot{a}^{(12)}_{65}$	
	$= 7.60$	$=(1.07)^{-30}(8.74)$	$=(1.07)^{-20}(8.74)$	
	$=7.60$	$= 1.1481$	$= 2.2586$	
PVB	91,200	68,889	81,309	$\Sigma = 241{,}398$
$\ddot{a}_{x:\overline{65-x}\rceil}$	—	$\ddot{a}_{\overline{30}\rceil} = 13.2777$	$\ddot{a}_{\overline{20}\rceil} = 11.3356$	avg $= 24.6133/2 = 12.3066$

$\therefore PVNC = 241{,}398 - 101{,}200 = 140{,}198$

$NC = 140{,}198 / 12.3066$
$= 11{,}392$

B

A key point is that the average temporary annuity is based on active participants only. It would be incorrect to include Smith with an annuity value of zero and to divide by 3!

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8. You are told that the non-investment G/L for 1989 is zero. This means that the investment G/L for 1989 must equal the total experience G/L for 1989:

$G/L = eUAL_1 - UAL_1$
$= eUAL_1 - 25{,}000$
$eUAL_1 = (1+i)(NC_0 + UAL_0) - (\text{Contrib} + \text{interest})$
$1.08\,(NC_0 + UAL_0) - 6000(1 + 9/12(.08))$

You must do the valuation at 1-1-89 to calculate both NC_0 and UAL_0. This problem is quite tricky since you have a normal retirement age of 63, with a salary scale and the projected unit credit method. Under PUC the accrued liability is defined as the present value of the "Funding accrued benefit". The "Funding accrued benefit" can be calculated using the benefit accrual formula for past service with projected salary.

1-1-90 Age 35 Past svc 10   1-1-89 Age 34 Past svc 9   1988 Pay 40,000 at age 33

Total service at age 63 = 38 years   Age 62 pay $= 40{,}000(1.04)^{29}$
$= 124{,}746$

1-1-89 FAB = .025(9 years) 124,746
$AL = .025(9)\,124{,}746\,(\ddot{a}_{63}^{(12)})\,D_{63}/D_{34}$
$= .025(9)\,124{,}746\,(8.582)(1.08)^{-29}$
$= 25{,}853$
1-1-90 FAB - 025(10 years) 124,746
$\Delta FAB = .025(124{,}746)$
1-1-89 $NC = .025(1)\,124{,}746\,(8.582)(1.08)^{-29}$
$= AL(1/9) = 2{,}873$

$eUAL_1 = 1.08(2{,}873 + 25{,}853) - 6000\,(1.06)$
$= 24{,}663$

D $G/L = 24{,}663 - 25{,}000 = 337$ loss since actual UAL exceeds expected

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9. Key item here is that Smith was originally included in the calculation of both the PVB and the average annuity. Must back out values as an active, then add in values for Smith as a retiree:

	Smith Active	Smith Retired
1-1-90 Age	55	55
Past service	18	18
Total svc	28	18
Proj Benefit	12(15)(28)	12(15)(18)[1-.06(5)-.03(5)]
	= 5040	= 1782
PV factor	${}_{10|}\ddot{a}^{(12)}_{65}$	$\ddot{a}^{(12)}_{55}$
	$=(1.085)^{-10}8.33$	= 12.33
PV Benefit	19,446	21,972
$\ddot{a}_{x:\overline{65-x}|}$	$\ddot{a}_{\overline{10}|} = 7.2469$	

	Valuation Incl Smith-Active	Valuation Excl Smith	Valuation Incl Smith Retd
PVB	662,000	-19,446	+ 21,972
UAL	163,250		
AAV	142,500		
PVNC	356,250 -19,446 = 336,804 + 21,972 = 358,776		
Average NC	195.34		
PVNC/Avg NC = $\Sigma\ddot{a}_{x:\overline{65-x}|}$	1823.74	-7.2469	= 1816.49
# Active ees	150	-1	= 149
Average $\ddot{a}_{x:\overline{65-x}|}$	12.1583		12.1912
$\frac{PVNC}{Avg\ \ddot{a}_{x:\overline{65-x}|}}$ = Total NC	29,301		29,429

(C) Increase in NC is 29,429 - 29,301 = 128

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10. Beware that there are different numbers of participants at each age. Must be careful in calculating average annuity for normal cost. Since we are given D_x values, use them in calculating PVB and $\ddot{a}_{x:\overline{65-x}|}$.

$N_{65}^{(12)} = N_{65} - \frac{11}{24} D_{65}$

$= 8872 - \frac{11}{24}(965)$

$= 8429.71$

	Active	Active	Retired	
1-1-90 Age	45	55	65	
# ees	15	10	5	
Service	20	30	40	
Total svc	40	40	40	
Projected ben	4800	4800	4800	
PV factor	$N_{65}^{(12)}/D_{45}$ $= 1.8617$	$N_{65}^{(12)}/D_{55}$ 3.8545	$N_{65}^{(12)}/D_{65}$ 8.7354	
Total PVB	134,041	185,014	209,651	$\Sigma = 528,706$
$\ddot{a}_{x:\overline{65-x}|}$	$(N_{45}-N_{65})/D_{45}$ $= 10.8858$	$(N_{55}-N_{65})/D_{55}$ $= 7.1829$	$\cancel{0}$	
Total annuity	163.2873	71.8290		$\Sigma = 235.1163$
# active ees	15	10		$\Sigma = 25$

$PVNC = PVB - AAV$
$= 528,706 - 300,000$
$= 228,706$

Avg annuity $= \frac{235.1163}{25}$
$= 9.4047$

$NC = \frac{228,706}{9.4047}$
$= 24,318$

(C)

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11. EANC is defined as $\frac{PVB_{EA}}{PVL_{EA}}$ when benefits are expressed as a dollar amount not related to pay. The accrued liability can be expressed as either a prospective or retrospective definition.

I. This is the retrospective definition of the AL: true

$$AL_{CA} = \sum_{t=EA}^{CA-1} NC_{EA} \frac{D_t}{D_{CA}} = EANC\left(\frac{N_{EA}-N_{CA}}{D_{CA}}\right)$$

II. This is the prospective definition of the AL: true

$$AL = PVB - PVNC$$
$$= B\left(\frac{N_{65}^{(12)}}{D_x}\right) - P\left(\frac{N_x - N_{65}}{D_x}\right)$$

III. This expression reflects substitution of the EANC based on benefit B in the prospective definition

$$P = \frac{PVB_{30}}{PVL_{30}} = B\frac{N_{65}^{(12)}}{D_{30}} \Big/ \left(\frac{N_{30}-N_{65}}{D_{30}}\right) = \frac{B \cdot N_{65}^{(12)}}{N_{30}-N_{65}}$$

$$AL = B\left(\frac{N_{65}^{(12)}}{D_{45}}\right) - \frac{B\,N_{65}^{(12)}}{N_{30}-N_{65}}\left(\frac{N_{45}-N_{65}}{D_{45}}\right)$$
$$= B\left(\frac{N_{65}^{(12)}}{D_{45}}\right)\left[1 - \frac{N_{45}-N_{65}}{N_{30}-N_{65}}\right]$$
$$= B\left(\frac{N_{65}^{(12)}}{D_{45}}\right)\left[\frac{N_{30}-N_{45}}{N_{30}-N_{65}}\right]$$

true

D ∴ All three are true

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12. The benefit defined as 50% of final year's comp should use pay at age 64. You are not given an S_x value at age 64, which seems to present a real problem. However, analysis of the S_x/S_{65} column reveals that the salary scale is 5% at every age. The previous problem gave various formulas for AL, but those were based on a normal cost expressed as a level dollar amount - do not be misled! The easiest way should be the retrospective defn:

$$EANC\% = \frac{PVB_{EA}}{PVE_{EA}} \qquad AL_x = (EANC\%)(Pay_x)\left(\frac{{}^sN_{EA} - {}^sN_x}{{}^sD_x}\right)$$

1-1-90 Age 35 Service 10 Entry Age 25

1989 Pay 25,000 at age 34 $\Rightarrow$ Age 64 Pay $= 25{,}000(1.05)^{30}$

$= 108{,}049$

Proj Benefit $= 54{,}024$

$PVB_{25} = 54{,}024\ (\ddot{a}_{65}^{(12)})\, D_{65}/D_{25}$

$= 54{,}024\ (8.736)\ 94{,}414\ /1{,}779{,}168$

$= 25{,}045$

$Pay_{25} = 25{,}000\ (1.05)^{-9} = 16{,}115$

$PVE_{25} = 16{,}115\ ({}^sN_{25} - {}^sN_{65})/{}^sD_{25}$

$= 16{,}115\ (193{,}660{,}240 - 30{,}013{,}858)/6{,}024{,}894$

$= 437{,}717$

$EANC\% = 25{,}045\ /437{,}717 = 5.72\%$

$EANC_{35} = 5.72\%\ (Pay_{35}) = .0572\,(25{,}000)(1.05) = 1{,}502$

$AL_{35} = EANC_{35}\ ({}^sN_{25} - {}^sN_{35})/{}^sD_{35}$

$= 1{,}502\ (193{,}660{,}240 - 138{,}500{,}016)/4{,}932{,}364$

$= 16{,}797$

B

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13. You must write down expressions for each annuity, and equate them to solve for P. The second annuity is easily written down using reversionary annuities

$$(P+500)\left(\ddot{a}_{60}^{(12)} - \frac{D_{62}}{D_{60}}\ddot{a}_{62}^{(12)}\right) + P\left(\frac{D_{62}\,\ddot{a}_{62}^{(12)}}{D_{60}}\right) = \text{Option A}$$ 1st 2yrs plus later

OR

$$P\ddot{a}_{60}^{(12)} + 500\left(\ddot{a}_{60}^{(12)} - \frac{D_{62}}{D_{60}}\ddot{a}_{62}^{(12)}\right) = \text{Option A}$$ life plus temporary

$$\text{Option B} = 500\,\ddot{a}_{60:60}^{(12)} + (500-.5P)\left[\ddot{a}_{60}^{(12)} - \ddot{a}_{60:60}^{(12)}\right] + (500-.5P)\left[\ddot{a}_{60}^{(12)} - \ddot{a}_{60:60}^{(12)}\right]$$ Joint-both plus either reversionary

$$= 500\,\ddot{a}_{60}^{(12)} + (500-.5P)\left[\ddot{a}_{60}^{(12)} - \ddot{a}_{60:60}^{(12)}\right]$$ if partic dies

$$-.5P\left[\ddot{a}_{60}^{(12)} - \ddot{a}_{60:60}^{(12)}\right]$$ if spouse dies

annuity for participant

$$\therefore\ P\ddot{a}_{60}^{(12)} + 500\,(9.815 - 125.296(9.394)/147.804)$$

$$= 500\,\ddot{a}_{60}^{(12)} + (500-P)\left[\ddot{a}_{60}^{(12)} - \ddot{a}_{60:60}^{(12)}\right]$$

$$P\ddot{a}_{60}^{(12)} + P\ddot{a}_{60}^{(12)} - P\ddot{a}_{60:60}^{(12)} = 500\,\ddot{a}_{60}^{(12)} + 500\,\ddot{a}_{60}^{(12)} - 500\,\ddot{a}_{60:60}^{(12)} - 925.7721$$

$$P = 500 - \frac{925.7721}{2\ddot{a}_{60}^{(12)} - \ddot{a}_{60:60}^{(12)}}$$

$$= 500 - \frac{925.7721}{2(9.815) - 8.094}$$

$$= 419.75$$

this is inside the implied range of 340 to 440

A

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14. You are not given values for $N_x^{(12)}$ or $N_{x:x-3}^{(12)}$, so you have to derive these as $N_x - 11/24 D_x$ and $N_{x:x-3} - 11/24\, D_{x:x-3}$ respectively. The spouse's benefit after the participant's death is 50% of the actuarially reduced benefit.

$$1000 \frac{N_{65}^{(12)}}{D_{55}} = B\left(\frac{N_{55}^{(12)}}{D_{55}}\right) + .5B\left(\frac{N_{52}^{(12)}}{D_{52}} - \frac{N_{55:52}^{(12)}}{D_{55:52}}\right)$$

$$B = \frac{1000\,(N_{65} - 11/24\, D_{65}) / D_{55}}{\frac{N_{55}}{D_{55}} - 11/24 + .5\left[\frac{N_{52}}{D_{52}} - \frac{11}{24} - \frac{N_{55:52}}{D_{55:52}} + \frac{11}{24}\right]}$$

$$= \frac{1000\left[95 - \frac{11}{24}(95-85)\right] / (263-240)}{\frac{263}{263-240} - \frac{11}{24} + .50\left[\frac{345}{345-316} - \frac{2168}{2168-1951}\right]}$$

$$= \frac{3931}{10.9764 + .5\,(11.8966 - 9.9908)}$$

$$= 329.54$$

$$.5B = 164.77$$

(B)

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15. Simple interest would reflect fractional years from each cash flow date to the end of the year. Since you don't know the interest rate assumed, simply calculate the allocation weights

Plan A: $100{,}000 + \frac{3}{4}(20{,}000) + \frac{2}{4}(20{,}000 - 10{,}000) + \frac{1}{4}(20{,}000) = 125{,}000$

Plan B: $60{,}000 + \frac{3}{4}(40{,}000) + \frac{2}{4}(0 - 15{,}000) + \frac{1}{4}(40{,}000) = 92{,}500$

Plan C: $\emptyset + \frac{3}{4}(100{,}000) + \frac{2}{4}(\emptyset) + \frac{1}{4}(\emptyset) = 75{,}000$

$\Sigma = 292{,}500$

Investment income allocated to Plan C $= 100{,}000\left(\frac{75{,}000}{292{,}500}\right)$

$= 25{,}641$

1/1/90 asset value for Plan C $= 300{,}000 - 5{,}000 + 25{,}641 = 320{,}641$

A

This is within the implied range of 308,000 to 320,750

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16. The calculation of both the PVB and PVE must reflect the 50% decrement at age 62:

	RA 62	RA 65	
1-1-90 Age	56	56	1989 pay 100,000 age 55
Past Service	1	1	
Total service	7	10	
RA - 1	61	64	
Pay at RA-1	$100{,}000(1.07)^6 = 150{,}073$	$100{,}000(1.07)^9 = 183{,}846$	
Proj Benefit	$.01(7)(150{,}073) = 10{,}505$	$.01(10)(183{,}846) = 18{,}385$	
PVB at age 56	$10{,}505\, v^6 \ddot{a}_{62}^{(12)} = \dfrac{10{,}505(9.394)}{(1.07)^6} = 65{,}758$	$18{,}385\, v^9 \ddot{a}_{65}^{(12)} = \dfrac{18{,}385(\cancel{9.394})\ 8.736}{(1.07)^9} = 87{,}360$	

$$PVNC = PVB - AAV$$
$$= .5(65{,}758) + (1-.5)87{,}360$$
$$= 76{,}559$$

$$PVE = 100{,}000(1.07)\Big(\underbrace{1 + \tfrac{1.07}{1.07} + \cdots + \left(\tfrac{1.07}{1.07}\right)^5}_{\text{Annuity based on retirement at age 62}} + .5\left(\tfrac{1.07}{1.07}\right)^6 + .5\left(\tfrac{1.07}{1.07}\right)^7 + .5\left(\tfrac{1.07}{1.07}\right)^8\Big)$$

56 57 61 62 63 64

$$= 107{,}000\,(6 + .5(3))$$
$$= 107{,}000\,(.5)(62-56 + 65-56) \quad \text{Combination of annuities if retire at 62 or 65}$$
$$= 802{,}500$$

$$NC\% = 76{,}559 / 802{,}500 = 9.54\%$$

(D) $$\text{1-1-90 } NC = 9.54\%\,(107{,}000) = 10{,}208$$

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17. Since you are told that the investment G/L is zero, you can calculate the experience G/L using the formula for non-investment G/L:

non-inv G/L = $eAL_1 - AL_1$

$eAL_1 = (1+i)(NC_0 + AL_0) - (\text{actual } BP + i)$

$= 1.07(2{,}000 + 21{,}000) = 24{,}610$

You can ignore the term for benefit payments since the participant is still active at 1-1-90. The definition of the accrued liability under the projected unit credit method is the present value of the "Funding Accrued Benefit". The "Funding Accrued Benefit" can be calculating using the benefit accrual rate for past service, with earnings projected to retirement age:

FAB = 1%(10 years)(Pay at 64)

1-1-90 Age 50
Service 10

1989 pay 40,000 at age 49
age 64 pay $= 40{,}000\,(1.05)^{15} = 83{,}157$

FAB = .01(10)(83157) = 8316

$AL = 8{,}316 \dfrac{N_{65}^{(12)}}{D_{50}} = 8{,}316\,(849/320) = 22{,}063$

(C) $\therefore$ Gain = 24,610 − 22,063 = 2,547 since actual AL is less than expected

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(17) Continued

The problem is defective, since the normal cost and accrued liability at 1-1-89 are inconsistent. This participant has 9 years of service at 1-1-89, so the normal cost should be 1/9 of the accrued liability:

$$1\text{-}1\text{-}89 \quad AL = \underbrace{.01(9)(\text{Proj FAP})}_{FAB}\, N_{65}^{(12)}/D_{49}$$

$$NC = .01(1)(\text{Proj FAP})\, N_{65}^{(12)}/D_{49}$$

This can get you into trouble if you try to calculate the G/L by source, then add the pieces for the total G/L:

Investment G/L = zero, since this is given!

Total G/L = salary G/L plus mortality G/L

For participant who survives, mortality loss is equal to $q_x AL_1 = AL_1 - p_x(AL_1)$

$$1\text{-}1\text{-}90\ AL = .01(10)(\text{Proj FAP})\, N_{65}^{(12)}/D_{50} \quad \text{[using same FAP as 1-1-89]}$$
$$= (10/9)(21{,}000)(D_{49}/D_{50})$$
$$= 25{,}083$$

Mortality loss $= 25083(q_{49}) = 25083(.004651) = 117$

$$\frac{D_{50}}{D_{49}} = v\,p_{49} \Rightarrow (1+i)\frac{D_{50}}{D_{49}} = p_{49} \Rightarrow q_{49} = 1 - 1.07(320/344)$$
$$= .004651$$

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(17) Continued

Salary scale G/L = 25083 − AL using \$40,000 pay for age 49

1-1-90 AL = .01(10)(83157)(849/320) as previously calculated
= 22,063

∴ salary scale gain = 3020
mortality loss = 117
net gain = 2903 not same as 2547 in orig solution

To get consistent results for the total G/L, you must force the Normal cost to be consistent with the accrued liability at 1-1-89:

Assume 1-1-89 NC = (1/9) 21,000 = 2333

Then eAL_1 = 1.07 (21,000 + 2333) = 24,966
AL_1 = (as previously calc) 22,063
2903 total G/L, which matches G/L by source!

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18. The definition of the Entry Age normal cost is $\frac{PVB_{EA}}{PVE_{EA}}$ EANC%. The current year's normal cost is based on this percentage multiplied by this year's pay.

1-1-90 Age 50   Entry Age 45   1989 Pay 25,000 age 49
Past Service 5   Total Svc 20   Age 64 pay $25{,}000(1.05)^{15}$
$= 51{,}973$

$$\text{Projected Benefit} = .01(20)(51{,}973) = 10{,}395$$

$$PVB_{EA} = 10{,}395\ \ddot{a}_{65}^{(12)}\ \frac{D_{65}}{D_{45}}$$
$$= 10{,}395\,(8.735)(1.07)^{-20}$$
$$= 23{,}464$$

$$PVE_{EA} = 25{,}000\,(1.05)^{-4}\left[\underset{\text{age }45}{1} + \underset{\text{age }46}{v\,p_{45}(1.05)} + \underset{\text{age }47}{v^2\,{}_2p_{45}(1.05)^2} + \dots + \underset{\text{age }64}{v^{19}\,{}_{19}p_{45}(1.05)^{19}}\right]$$
$$= 20{,}568\left[1 + \frac{1.05}{1.07} + \left(\frac{1.05}{1.07}\right)^2 + \dots + \left(\frac{1.05}{1.07}\right)^{19}\right]$$
$$= 20{,}568\ \ddot{a}_{\overline{20|}\,1.9\%}$$
$$= 345{,}886$$

$$EANC\% = \frac{23{,}464}{345{,}886} = 6.78\%$$

$$\text{1-1-90 } EANC = 6.78\%\ (1990 \text{ pay at age } 50)$$
$$= 6.78\%\ (25000)(1.05)$$
$$= 1781$$

D

With all the salary scale manipulation, many chances for error. Last chance to go wrong is using 25,000 for 1990.

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19. Under the ILP method, each new layer of benefit is funded prospectively. For this participant, the original layer of normal cost is calculated at 1-1-85, and a second layer is calculated at 1-1-88. The accrued liability can be calculated retrospectively.

1-1-85 Age 25
Past Service $\emptyset$
Total Service 40
Total Pay 42,000
Projected Ben 33,600 = 42,000 (.02)(40)

$$PVB = 33{,}600\, \ddot{a}_{65}^{(12)}\, D_{65}/D_{25}$$
$$= 33{,}600\,(9.345)\, v^{40}$$
$$\text{1-1-85 NC} = \frac{33{,}600\,(9.345)\, v^{40}}{\ddot{a}_{\overline{40}|.07}}$$
$$= \frac{33{,}600\,(9.345)}{\ddot{s}_{\overline{40}|.07}} = 1470$$

1-1-88 Age 28
Total Pay 48,000
Projected Ben 38,400 = 48,000 (.02)(40)
Δ PB 4,800 = 6,000 (.02)(40)

$$\text{1-1-88 } \Delta NC = \frac{4800\,(9.345)}{\ddot{s}_{\overline{37}|.07}} = 261$$

1-1-90 AL = accum value of past normal costs
$$= 1470\,(\ddot{s}_{\overline{5}|.07}) + 261\,(\ddot{s}_{\overline{2}|.07})$$
$$= 9{,}045 + 579$$
$$= 9{,}624$$

(B)

Time line for accrued liability – does not include 1-1-90 NC

1-1-85	1-1-86	1-1-87	1-1-88	1-1-89	1-1-90
1470	1470	1470	1470 261	1470 261	⊛

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20. This problem is a reversal of what has been asked in the past. The normal form for actives is being de-liberalized from a 100% J+S to a life annuity. Offsetting this is an increase in the retirement benefit; must determine PVB for actives!

1-1-90 Valuation before amendments

PVE/E · 4,850 / 375 = 12.9333
PVNC - NC (12.9333) = 94,000 (12.9333) = 1,215,733
Total PVB = PVNC + AAV = 1,215,733 + 730,000 = 1,945,733
Actives PVB - 1,945,733 − Inactives 500,000 · 1,445,733

The weighted PV factor for the actives is based on 15% receiving a life annuity and 85% receiving the J+S:

$.15(\ddot{a}_{65}^{(12)}) + .85(\ddot{a}_{65}^{(12)} + \ddot{a}_{65}^{(12)} - \ddot{a}_{65:65}^{(12)})$
$= .15(10.00) + .85(10.0 + 10.0 - 8.2)$
$= 11.53$

As a check, this should equal the PV factor at age 65 for an 85% joint and survivor annuity:

$\ddot{a}_{65}^{(12)} + .85(\ddot{a}_{65}^{(12)} - \ddot{a}_{65:65}^{(12)}) = 10 + .85(10 - 8.2) = 11.53$ ✓

1-1-90 Valuation after amendments

The PV factor at age 65 is only based on a life annuity for all active employees. The resulting liability is

(10/11.53) 1,445,733 = 1,253,888

After the benefit increase, the active liability doubles

Total PVB = 500,000 Inactives + 2(1,253,888) = 3,007,777

(B) NC = (3,007,777 − 730,000 AAV) / 12.9333 = 176,117