



SoftwarePolish

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SPRING 1988 EA-1B EXAM SOLUTIONS

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Revision History:

05/02/97	Improved problem 7 -	added column headings
05/02/97	Corrected problem 10 -	removed reference to UAL
05/02/97	Corrected problem 13 -	clarified normal cost calculation, showed retrospective calcs
05/02/97	Improved problem 19 -	clarified traditional unit credit versus projected unit credit

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Problem 1 - Page 1

Under the Individual Level Premium cost method, benefit changes are funded prospectively from the date of change. When the projected benefit under the plan changes, either due to plan amendments or salary changes, a new layer of normal cost is created.

In this problem, we should calculate the initial normal cost based on amortization from the effective date of the plan. The 1988 normal cost will be that amount increased plus the additional layer of normal cost due to the increase in salary.

With no deaths or terminations prior to retirement, we discount liabilities prior to age 65 at interest only. We can use annuities certain for the temporary annuity to retirement age.

In order to calculate the experience gain, we apply the usual formula:

$$\text{total G/L} = {}_e\text{UAL}_1 - \text{UAL}_1.$$

Under the ILP method, the initial Accrued Liability is zero, and the expected UAL is also zero. Based on the retrospective definition of the Accrued Liability, we have

$$\text{AL}_1 = (D_{X-1} / D_X) (\text{NC}_0 + \text{AL}_0)$$

Since our assumptions use no pre-retirement decrements, and we know the initial Accrued Liability is zero under ILP, this simplifies to

$$\text{AL}_1 = 1.06 (\text{NC}_0)$$

01/01/87 valuation

Age 50

$$\begin{aligned} 1/1 \text{ ILP NC} &= .40(100,000) (v^{15}) (\ddot{a}_{65}^{(12)}) / \ddot{a}_{15} \cdot 1.06 \\ &= 40,000 (.4173) (9.333) / 10.2950 \\ &= 15,131 \end{aligned}$$

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Problem 1 - Page 2

01/01/88 valuation

Age 51

Change in projected benefit = $.40(120,000-100,000)$

$$\begin{aligned}\text{chg ILP NC} &= .40(20,000)(v^{14})(\ddot{a}_{65}^{(12)}) / \ddot{a}_{14}^{1.06} \\ &= 8,000(.4423)(9.333)/9.8527 \\ &= 3,352\end{aligned}$$

$$\begin{aligned}1/1 \text{ ILP NC} &= 15,131 + 3,352 \\ &= 18,483\end{aligned}$$

$$\begin{aligned}\text{UAL} = \text{AL} - \text{AAV} &= 1.06 (15,131) - 15,000 \\ &= 16,039 - 15,000 \\ &= 1,039\end{aligned}$$

Since the expected UAL is zero, this is a loss of 1,039. The final deposit as of 01/01/88 is the normal cost plus the loss amortized over ten years:

$$\begin{aligned}01/01/88 \text{ contrib} &= 18,483 + 1,039 / \ddot{a}_{10}^{1.06} \\ &= 18,617\end{aligned}$$

answer is B

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Problem 2

Under the Individual Aggregate method, the normal cost is calculated on an individual basis. The assets are allocated to each participant, and the PVNC is calculated directly as PVB - allocated AAV. With no deaths or terminations prior to retirement, we discount liabilities prior to age 65 at interest only. We can use annuities certain for the temporary annuity to retirement age.

The first step is to calculate the accrued benefit at 01/01/88 to allocate the assets:

	Smith	Brown	
current age	50	60	
hire age	45	45	
total service	20	20	
past service	5	15	
accrued benefit	600	1,800	
P.V. at age 65	6,000	18,000	
P.V. at 1/1/88	$6,000v^{15}$	$18,000v^5$	
	= 2,504	= 13,451	
alloc. assets	1,569	8,431	10,000
proj. benefit	2,400		
P.V. at age 65	24,000		
P.V. at 1/1/88	$24,000v^{15}$		
	10,014		
PVNC = PVB - AAV	8,445		
PVL/L = $\ddot{a}_{\overline{15} .06}$	10.2950		
Normal cost	820		

answer is D

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Problem 3

Attained Age Normal is an aggregate cost method. Since 01/01/88 is the effective date of the plan, you must use the Unit Credit method to establish the initial Accrued Liability.

	Smith	Brown	Total
current age	45	60	
hire age	30	40	
past service	15	20	
accrued benefit	1,800	2,400	
P.V. at age 65	16,200	21,600	
P.V. at 1/1/88	$16,200v^{20}$	$21,600v^5$	
Accrued Liability	5,051	16,141	21,192 (equals UAL)
future service	20	5	
future accruals	3,600	900	
proj. benefit	5,400	3,300	(past + future)
P.V. at age 65	48,600	29,700	
P.V. at 1/1/88	$48,600v^{20}$	$29,700v^5$	
	= 15,153	= 22,193	37,347
PVNC = PVB - AAV - UAL			16,155
$\ddot{a}_{\overline{n} .06}$	12.1581	4.4651	16.6232

$$\text{Average PVL/L} = 16.6232/2 = 8.3116$$

$$\text{Normal cost} = 16,155 / 8.3116 = 1,944$$

answer is C

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Problem 4

Frozen Initial Liability is an aggregate cost method, so the PVNC is defined as $PVB - AAV - UAL$. In this problem, we seem to be given all of these items, which would make this a very easy problem. The trick to this problem is that you must allow for the handling of employee contributions.

The one item that we are missing is the present value of future employee contributions. Since the contributions are defined as a uniform percentage of pay, you can calculate this value from the PVE:

$$PVEEC = .02 (12,500,000) = 250,000$$

ASSETS		LIABILITIES	
AAV	650,000	PVB	2,600,000
UAL	300,000		
PVEEC	250,000		
PVNC	1,400,000 (balancing item)		
	<hr/> 2,600,000		<hr/> 2,600,000

$$\begin{aligned} NC &= PVNC / (PVE/E) \\ &= 1,400,000 / 12.5 \\ &= 112,000 \text{ at } 01/01/88 \\ &= 112,000 (1.06) \text{ at } 12/31/88 = 118,720 \end{aligned}$$

The fact that this problem asks for the normal cost at the end of the year is a cheap trick - be sure to read the problem carefully.

answer is B

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Problem 5

This problem has salary scales, which really tests your ability to work carefully. Under the Entry Age Normal method, the salary scales complicate the calculations even more. We must calculate the normal cost percentage, which is defined as the PVB at entry age divided by the PV of salary at entry age.

With no deaths or terminations prior to retirement, we discount liabilities prior to age 65 at interest only. We can use annuities certain for the temporary annuity to retirement age. The steps in working this problem are: (i) calculate the final average pay, (ii) calculate the projected benefit, (iii) calculate the PVB at entry age, (iv) calculate the PV of pay at entry age, and (v) calculate the normal cost as a percentage of pay. This year's normal cost equals this percentage multiplied by this year's earnings.

Calculate the final average salary by projecting pay from current age to age 64, then multiplying by $\ddot{a}_{37}/3$:

1/1/88 valuation

Age = 50, Age 50 pay = 50,000

Entry age = 35

Age 64 pay = $50,000(1.05)^{14} = 98,997$

F AE3 = Age 64 pay * $\ddot{a}_{37}.05 / 3$

= 94,357

projected benefit = $.5(94,357) = 47,179$

$PVB_{ea} = 47,179 * \ddot{a}_{65}^{(12)} * v^{30}$

= $47,179(9.00)(.1741)$

= 73,929

To calculate the PVE, write down the expression with interest rates and salary scales. Then the expression can be evaluated as an annuity certain:

Entry age pay = $50,000(1.05)^{15} = 24,051$

PVE = $24,051 [1 + (1.05/1.06)^1 + \dots + (1.05/1.06)^{29}]$ (30 terms)

= $24,051 (\ddot{a}_{30}^{1.95\%})$

= $24,051 (26.2356) = 630,990$

EANC% = $73,929 / 630,990 = 11.72\%$

NC = $50,000 * .1172$

= 5,858

answer is B

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Problem 6

This end of the year valuation problem is designed to confuse you. The standard formula for expected UAL should be used:

$$\begin{aligned} {}_e\text{UAL}_1 &= (1+i)(\text{NC}_0 + \text{UAL}_0) - (\text{contrib} + \text{int}) \\ &= 1.06(20,000 + 250,000) - [7,000(1+.06(21/12)) + 7,000(1+.06(18/12)) \\ &\quad + 20,000(1+.06(12/12))] \\ &= 286,200 - (7,735 + 7,630 + 21,200) \\ &= 249,635 \end{aligned}$$

The key thing to realize is that the 1986 contributions would not be included in either the AAV or the UAL for the 12/31/86 valuation. The purpose of the 12/31/86 valuation is to determine the contribution required for the 1986 plan year.

answer is D

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Problem 7

Under the Frozen Initial Liability method, a change in plan benefits or assumptions creates a new layer of UAL. The layer is calculated as the effect of the change in benefits or assumptions on the Entry Age Normal accrued liability at the valuation date.

In this problem, you are given most of the items for the change in assumptions. You can easily determine the effect of the change in benefits by multiplying the \$10 results by 12/10:

	\$10.00 ARA 65	\$10.00 ARA 62	\$12.00 ARA 62
EAN accrued liability	13,100	16,350	19,620
Change in UAL (EAN AL)	-0-	3,250	3,270
Adjusted UAL	6,150	9,400	12,670
PV future benefits	15,000	18,400	22,080
Actuarial asset value	5,000	5,000	5,000
PVNC = PVB - UAL - AAV	3,850	4,000	4,410

With one participant, the average PVL/L is simply the value for that participant. Since there are no pre-retirement decrements, you can calculate an annuity due. The only real trick to this problem is to stop counting service years at age 62

$$1/1/88 \text{ age} = 50$$

$$\ddot{a}_{\overline{12}|.06} = 8.8869$$

$$\text{Normal cost} = 4,410 / 8.8869 = 496$$

answer is C

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Problem 8

Under the Aggregate method, the PVNC is calculated on an aggregate basis as PVB - AAV. The normal cost is calculated using the average temporary annuity to normal retirement age. In the 1-1-87 valuation, the Unit Credit cost method was used.

With no deaths or terminations prior to retirement, we discount liabilities prior to age 65 at interest only. We can use annuities certain for the temporary annuity to retirement age.

The only item needed to calculate the normal cost at 1-1-88 is the value of the monthly annuity at age 65. This can be calculated with the information given for the 1-1-87 valuation.

1-1-87 valuation - Unit Credit

Age = 47, Past svc = 9, Future svc = 18

Accrued benefit = $9(12)(25) = 2,700$

Accrued Liability = UAL + AAV = $2,500 + 6,300 = 8,800 =$
= P.V. of accd ben

$$= 2,700 * v^{18} * \ddot{a}_{65}^{(12)}$$

$$\ddot{a}_{65}^{(12)} = 8,800 / (.3503 * 2,700) = 9.303$$

1-1-88 valuation - Aggregate method

Age = 48, Past svc = 10, Total svc = 27

Projected benefit = $27(12)(25) = 8,100$

PV of proj ben = $8,100 * v^{17} * \ddot{a}_{65}^{(12)}$

$$= 8,100 * .3714 * 9.303$$

$$= 27,984$$

(Equals $8,800 * 1.06 * 3$)

AAV = $1.10(6,300) + 1,250 = 8,180$

PVNC = $27,984 - 8,180 = 19,804$

PVL/L = $\ddot{a}_{17}^{1.07} = 11.1059$

NC = $19,804 / 11.1059 = 1,783$

answer is B

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Problem 9

Under the Frozen Initial Liability cost method, the PVNC is defined as $PVB - AAV - UAL$. You are given enough information to calculate the AAV, and the UAL is calculated based on the standard formula:

$$e^{UAL_1} = (1+i)(NC_0 + UAL_0) - (\text{contrib} + \text{int})$$

To apply this formula, you have to calculate the UAL in the 1/1/87 valuation as $PVB - AAV - PVNC = 500,000 - 210,000 - 80,000 = 210,000$.

$$\begin{aligned} UAL_1 &= 1.06(25,000 + 210,000) - [1.03(25,000) + 20,000] \\ e^1 &= 249,100 - 45,750 = 203,350 \end{aligned}$$

$$\begin{aligned} AAV &= 1.084(80,000) + 1.042(25,000) + 20,000 \\ &= 132,770 \end{aligned}$$

$$PVNC = PVB - AAV - UAL = 565,000 - 132,770 - 203,350 = 228,880$$

answer is E

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Problem 10

The Aggregate method is an aggregate cost method. It is necessary to do the valuation with both Smith and Brown as active employees, then value Brown as a retired employee.

	Smith	Brown	Total
current age	55	62	
hire age	25	25	
total service	40	40	
proj. benefit	4,800	4,800	
P.V. at age 65	44,880	44,880	
D_{65}/D_x	178/367	178/224	
P.V. at 1/1/88	21,767	35,664	57,431
AGG PVNC = PVB - AAV			42,431
$(N_x - N_{65}) / D_x$	2,730/367	623/224	
	7.4387	2.7813	10.2200

Average PVL/L = $10.2200/2 = 5.1100$

Normal cost = $42,431 / 5.1100 = 8,304$

When Brown retires at 12/31/87, the present value of benefits is revised:

Brown's early ret benefit	$(37\text{yrs}) * (\$120) * (1 - 3(.06)) = 3640.80$
Brown's PV of benefits	$10.10 * 3640.80 = 36,772$
Total PVB	$21,767 + 36,772 = 58,539$
Revised PVNC	$58,539 - 15,000 = 43,539$
Revised normal cost	$43,539 / 7.4387 = 5,853$
Change in NC	$5,853 - 8,304 = -2,451$

answer is A

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Problem 11

This problem can be worked using the formula for non-investment G/L, since the plan assets do not affect the mortality G/L.

$$\text{non-inv G/L} = eAL_1 - AL_1$$

$$eAL_1 = (1+i)(NC_0 + AL_0) - (\text{actual ben pmts} + \text{interest})$$

Since this participant has retired, the normal cost in the preceding formula is zero. You must calculate the monthly benefits paid plus interest to the next valuation date.

$$AL_1 = 12,000 \ddot{a}_{\overline{51}|.06}^{(12)} = 52,176$$

$$\begin{aligned} AL_0 &= 12,000 \ddot{a}_{\overline{69:61}|.06}^{(12)} = 12,000 (\ddot{a}_{\overline{61}|.06}^{(12)} + N_{75}^{(12)} / D_{69}) \\ &= 12,000 (5.076 + 5,006 / 1,344) \\ &= 105,608 \end{aligned}$$

$$\begin{aligned} eAL_1 &= (1.06)(105,608) - [1,000 (12 + .06(12+11+\dots+1)/12)] \\ &= 111,944 - 12,000 (1.0325) \\ &= 99,554 \end{aligned}$$

$$G/L = 99,554 - 52,176 = 47,378$$

answer is D

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Problem 12

Attained Age Normal is an aggregate cost method. The unfunded actuarial liability is defined equal to the expected value:

$$\begin{aligned} {}_e\text{UAL}_1 &= (1+i)(\text{NC}_0 + \text{UAL}_0) - (\text{contrib} + \text{int}) \\ &= (1.06)(\text{NC}_0 + \text{UAL}_0) - (1.06)(\text{NC}_0 + 3,000) \\ &= (1.06)(\text{UAL}_0 - 3,000) \end{aligned}$$

Last year's UAL is equal to the initial accrued liability, since last year's valuation date is the plan effective date. The Unit Credit method was used at 1-1-87 to calculate the IAL; Unit Credit defines the accrued liability as the present value of the accrued benefit:

	Smith	Brown	Total
1-1-87 age	50	60	
hire age	25	45	
past service	25	15	
monthly accd ben	$20(15) + 5(20)$ 400	$15(15)$ 225	
P.V. at age 65	44,800	25,200	
P.V. at 1-1-87	$44,800v^{15}$	$25,200v^5$	
Accrued Liability	18,693	18,831	37,524

$$1-1-88 \text{ UAL} = 1.06(37,524 - 3,000) = 36,596$$

answer is B

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Problem 13 - Page 1

Under the aggregate variation of the Entry Age Normal cost method, the EANC percentage is still defined as the PVB at entry age divided by the PV of salaries at entry age. In this problem you would calculate the normal cost as a level dollar amount, since the benefit is not based on salaries.

Instead of doing this calculation separately for each participant, the numerator and denominator are the sums of these items for all participants. The accrued liability can be calculated on a prospective or a retrospective basis, but it should be done "in the aggregate", as shown below.

	Smith	Brown	Total
current age	35	45	
entry age	25	35	
total service	40	30	
proj. benefit	4,800	3,600	
PVB at age 65	44,880	33,660	
D_{65}/D_{ea}	189/2,441	189/1,348	
PVB at entry	3,475	4,719	8,194
$\ddot{a}_{ea:65-ea} 1.06$	15.62	14.22	29.84
Average annuity	29.84 / 2 = 14.92		
Normal cost	8,194 / 14.92 = 549.20		
D_{65}/D_x	189/1,348	189/737	
PVB at age x	6,293	8,632	14,924
$\ddot{a}_{x:65-x} 1.06$	14.22	11.84	26.06
Average annuity	26.06 / 2 = 13.03		
PV of normal cost	549.20 * 13.03 = 7,156		
AL = PVB - PVNC	14,924 - 7,156 = 7,768		

See the next page for the calculation of the accrued liability based on the retrospective definition.

answer is C

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Problem 13 - Page 2

Instead of doing this calculation prospectively, you can calculate the retrospective accrued liability based on $(N_{ea} - N_{ca})/D_{ca}$. This requires separate calculations of $N_{ea} - N_{ra}$ and $N_{ca} - N_{ra}$, then you can calculate the retrospective annuity due.

	Smith	Brown	Total
current age	35	45	
entry age	25	35	
$N_{ea} - N_{65}$	$2,441 * 15.62$ = 38,128	$1,348 * 14.22$ = 19,169	
$N_{ca} - N_{65}$	$1,348 * 14.22$ = 19,169	$743 * 11.84$ = 8,726	
$N_{ea} - N_{ca}$	18,960	10,442	
Annuity due	$18,960 / 1,348$ = 14.07	$10,442 / 737$ = 14.17	
Average annuity	$28.24 / 2$	= 14.12	
Retrospective AL	$549.20 * 14.12$	= 7,755	

Note that a slightly different answer results from the retrospective approach, but it still falls in the same range.

answer is C

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Problem 14

This problem can be worked using the formula for non-investment G/L, since the plan assets do not affect the mortality G/L. This problem can be worked in several ways, but the standard formula is less susceptible to error.

$$\text{non-inv G/L} = {}_eAL_1 - AL_1$$

$${}_eAL_1 = (1+i)(NC_0 + AL_0) - (\text{actual ben pmts} + \text{interest})$$

$$NC_0 = (1\text{yr})(120)(9.35)(D_{65}/D_{45}) \quad \text{per participant}$$

$$AL_0 = (7\text{yr})(120)(9.35)(77/303) \quad \text{per participant}$$

$$NC_0 + AL_0 = 2,281 \quad \text{per participant}$$

$${}_eAL_1 = (1,000 \text{ ees})(1.06)(2,281) = 2,417,891$$

$$AL_1 = (8\text{yr})(120)(9.35)(77/283) \quad \text{per participant}$$

$$= 2,442 \quad (\text{should equal } 2,281 * 303/283)$$

$$AL_1 = (988 \text{ ees})(2,442) = 2,412,926$$

$$G/L = 2,417,891 - 2,412,926 = 4,965$$

answer is E

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Problem 15 - Page 1

This problem requires an understanding of how to reflect experience in bringing one year's valuation results forward to the next year. With no pre-retirement decrements, the present value of benefits simply grows at interest each year if all assumptions are met. With no participants within five years of assumed retirement age, we know that all of the participants are included in the valuation results in both years, and there were no benefit payments in 1987.

In this problem, the salaries grew at 7.5%, so the projected benefit would actually increase based on the ratio $(1.075/1.04)$. Based on the information given, the fund can be brought forward with 6% interest.

1-1-87 Valuation Balance Sheet: FIL Method

UAL	500,000	
AAV	100,000	
PVNC	1,400,000	balancing item
		Normal cost % = $84,000 / 600,000$
		= 14.00% of pay
PVB	2,000,000	

Since the PVE can be expressed as

$$\text{EARN} * (1 + [(1+s)/(1+i)]^1 + [(1+s)/(1+i)]^2 + \dots),$$

the expected PVE_1 is $(1+i) * (PVE_0 - \text{EARN}_0)$.

1-1-88 Valuation Balance Sheet

UAL	469,040	=	$1.06(500,000 \text{ UAL} + 84,000 \text{ normal cost}) - 150,000$
AAV	256,000	=	$1.06(100,000 \text{ AAV} + 150,000 \text{ contribution})$
PVB	2,191,346	=	$1.06(2,000,000) (1.075/1.04)$ (reflect interest and salary scale)
PVNC	1,466,306	=	$PVB - AAV - UAL$
PVE	10,299,327	=	$1.06(10,000,000 - 600,000) (1.075/1.04)$
EARN	645,000	=	$1.075(600,000)$
PVE/E	15.9679	=	$10,299,327 / 645,000$
NC	91,828	=	$1,466,306 / 15.9679$

answer is B

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Problem 15 - Page 2

If you are unsure of this approach, you can do some extra work and set up the normal cost percentage for the expected balance sheet at 1-1-88:

1-1-88 Expected Balance Sheet

$$\text{PVB } 2,120,000 = 1.06(2,000,000)$$

(reflect interest and salary scale)

$$\text{PVNC } 1,394,960 = \text{PVB} - \text{AAV} - \text{UAL} = 2,120,000 - 256,000 - 469,040$$

$$\text{PVE } 9,964,000 = 1.06(10,000,000 - 600,000)$$

$$\text{EARN } 624,000 = 1.04(600,000)$$

$$\text{PVE/E } 15.9679 = 9,964,000 / 624,000$$

$$\text{NC } 87,360 = 1,394,960 / 15.9679 \quad \text{NC \%} = 87,360 / 624,000 = 14.00$$

Since you get exactly the same normal cost %, this approach may be valid.

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Problem 16

One way to miss this problem is to try to take the change in cost method into account! Both the 1987 and 1988 valuations are based on the FIL method. The UAL at 1-1-87 is calculated as the EAN accrued liability less the AAV at 1-1-87.

You must calculate the normal cost for 1987 in order to write down the UAL at 1-1-88. Then you must calculate the normal cost as a percentage of pay at 1-1-88.

1-1-87 Valuation Balance Sheet

UAL	3,500,000	=	6,500,000	-	3,000,000
PVNC	4,500,000	=	11,000,000	-	3,500,000 - 3,000,000
PVE/E	6.8182	=	112,500,000	/	16,500,000
NC	660,000	=	4,500,000	/	6.8182

1-1-88 Valuation Balance Sheet

$$\begin{aligned} eUAL_1 &= (1+i)(NC_0 + UAL_0) - (\text{contrib} + \text{int}) \\ &= 1.06(660,000 + 3,500,000) - 1,000,000 \\ &= 3,409,600 \\ PVNC &= PVB - UAL - AAV = 12,000,000 - 3,409,600 - 4,000,000 \\ &= 4,590,400 \\ NC \% &= PVNC / PVE = 4,590,400 / 115,000,000 \\ &= 3.992\% \end{aligned}$$

answer is D

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Problem 17

Under the Attained Age Normal method, the UAL is written down each year based on the formula for the expected UAL. Since Brown was hired on 1-1-88, you should ignore him in calculating the UAL. The initial UAL is defined based on the Unit Credit method. You must calculate the present value of Smith's accrued benefit at 1-1-87 as the IAL. The valuation at 1-1-87 must be performed to calculate the normal cost in order to write down the UAL from 1-1-87 to 1-1-88.

Smith	
1-1-87 age	44
hire age	30
past service	14
accrued benefit	1,680
P.V. at age 65	16,800
P.V. at 1/1/87	16,800v ²¹
Accrued Liability	4,942
total service	35
proj. benefit	4,200
P.V. at 1/1/87	12,355 = 4,942 (4,200 / 1,680)
PVNC	= PVB - AAV - UAL
	= 12,355 - 0 - 4,942
	= 7,413
$\ddot{a}_{\overline{21} 1.06}$	= 12.4699
NC	= 7,413 / 12.4699 = 594
1-1-88 UAL	= (1+i)(NC ₀ +UAL ₀) - (contrib+int)
	= 1.06(594 + 4,942) - 1,000
	= 4,869

answer is B

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Problem 18

This problem can be worked using the formula for non-investment G/L, since the plan assets do not affect the mortality G/L. This problem can be worked in several ways, but the standard formula is less susceptible to error.

$$\text{non-inv G/L} = {}_eAL_1 - AL_1$$

$${}_eAL_1 = (1+i)(NC_0 + AL_0) - (\text{actual ben pmts} + \text{interest})$$

The actual accrued liability at 1/1/88 is simply a 12,000 life annuity. In the formula above, there will be no normal cost since Smith retired.

$$AL_1 = (12,000)\ddot{a}_{66} = 103,200$$

$$\begin{aligned} {}_eAL_0 &= (1+i)(12,000)[\ddot{a}_{65} + \tfrac{1}{2}(\ddot{a}_{60} - \ddot{a}_{60:65})] - (\text{actual BP} + \text{int}) \\ &= 1.06(12,000)[8.95 + .5(10.15 - 8.05)] - 1.06(12,000) \\ &= 1.06(12,000)(10.0) - 1.06(12,000) \\ &= 114,480 \end{aligned}$$

$$\text{non-inv G/L} = {}_eAL_1 - AL_1 = 114,480 - 103,200 = 11,280$$

answer is B

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Problem 19

Under the Projected Unit Credit method, the accrued liability is defined as the present value of the "funding" accrued benefit. The formal definition of the "funding" accrued benefit is that it is based on a pro-rata share of the projected benefit. If the plan has different rates of benefit accrual (including a cap on credited service), then you must reflect the different rates of benefit accrual in the pro-rata adjustment. Since this plan accrues benefits at a uniform rate over all years of service, the "funding" accrued benefit is determined with a simple service pro-rate.

Under the short cut approach to projected unit credit, the "funding" accrued benefit is determined by applying the benefit formula based on past service to the projected final average earnings at retirement age. With no pay related benefits, and no salary scale, this problem will give identical results under projected unit credit as under the traditional unit credit method.

This problem is very tricky for several reasons. If you use projected unit credit, you must be sure to handle the pro-rata adjustments of the benefits based on the different retirement ages. You must also calculate the present value factors based on the correct retirement ages. This is probably the most difficult question on the exam.

Old plan, old assumptions

1-1-88 age	50	
hire age	30	
past service	20	
future service	15	(age 65)
total service	35	

Accrued benefit	4,800	= 20 * 12 * 20
P.V. at 1-1-88	10,455	= 4,800 * (440 / 202)
Accrued liability	10,455	

New plan, New assumptions

1-1-88 age	50	
hire age	30	
past service	20	
future service	10	(age 60)
total service	30	

Accrued ret ben	4,800	= 20 * 12 * 20
P.V. ret ben	18,701	= 4,800 * (787 / 202)
Accd supplement	1,600	= 6.6667 * 12 * 20
P.V. supplement	2,749	= 1,600 * (787 - 440) / 202
Accrued liability	21,450	= 2,749 + 18,701
Change in accd liab	10,995	= 21,450 - 10,455

answer is C

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Problem 20

You must derive a relationship between successive years' normal cost values under Unit Credit.

$$1-1-87 \text{ NC} = (100\text{ees})(120) \left(N_{65}^{(12)} / D_{60} \right)$$

If all participants survived from 1-1-87 to 1-1-88 we would have

$$1-1-88 \text{ NC} = (100\text{ees})(120) \left(N_{65}^{(12)} / D_{61} \right)$$

Since only p_{60} participants are expected to survive, we have

$$1-1-88 \text{ NC} = (p_{60})(100\text{ees})(120) \left(N_{65}^{(12)} / D_{61} \right)$$

The ratio of 1-1-88 normal cost to 1-1-87 is

$$\text{Ratio} = p_{60} * D_{60} / D_{61} = p_{60} * (1+i) / p_{60} = 1.06$$

The 1-1-88 normal cost equals $1.06(100,000) = 106,000$

answer is D