



SoftwarePolish

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SPRING 2001 EA-1 EXAM SOLUTIONS

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- 1 The key to this problem is knowing the basic relationships for discounting and compounding interest:

$$\left[1 + \frac{i^{(m)}}{m}\right]^m = 1 + i = \left[1 - \frac{d^{(m)}}{m}\right]^{-m}$$

You can use the information given to set up expressions for compounding periods of both m and $2m$, then solve for m :

$$1 + i = \left[1 - \frac{d^{(m)}}{m}\right]^{-m} = 1 + i = \left[1 - \frac{d^{(2m)}}{2m}\right]^{-2m}$$

$$1 + i = \left[1 - \frac{.085256}{m}\right]^{-m} = \left[1 - \frac{.085715}{2m}\right]^{-2m}$$

$$\left[1 - \frac{.085256}{m}\right]^{-m} = \left[1 - \frac{.085715}{2m}\right]^{-m} \left[1 - \frac{.085715}{2m}\right]^{-m}$$

Now take the m^{th} root of both sides of the equation, and multiply both sides by $4m^2$; and invert both sides

$$1 - \frac{.085256}{m} = \left(1 - \frac{.085715}{2m}\right) \left(1 - \frac{.085715}{2m}\right)$$

$$4m(m - .085256) = (2m - .085715)^2$$

$$4m^2 - 4(.085256)m = 4m^2 - 4(.085715)m + (.085715)^2$$

$$.00184m = .00735$$

$$m = 4.002 \quad \text{call it } 4 \Rightarrow \text{quarterly}$$

$$1 + i = \left[1 - \frac{.085256}{4}\right]^{-4} = 1.0900$$

$$= \left[1 + \left(\frac{i^{(12)}}{12}\right)\right]^{12}$$

$$i^{(12)} = \left[(1.09)^{\frac{1}{12}} - 1\right] \cdot 12$$

$$= 8.649\%$$

$$1000 i^{(12)} = 86.488$$

(C)

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- 2 This is basically a trial and error problem. The reason is that you can't easily solve this expression for the value of m :

$$1800 \left[1 + \frac{.15}{m} \right]^{2m} = 2420.80$$

The best approach is to try the middle answer, and decide whether to increase or decrease the compounding period. With quarterly compounding, the value after 2 years is

$$1800 \left[1 + \frac{.15}{4} \right]^8 = 2416.45$$

You want a slightly larger value, so the next best guess is compounding every 2 months, or six times per annum:

$$1800 \left[1 + \frac{.15}{6} \right]^{12} = 2420.80 \quad \textcircled{B}$$

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(3) Now you can solve for the final smaller amount:

$$7264.89 (a_{\overline{25}|1.06}) + Xv^{26} = 93,472$$

$$Xv^{26} = 93,472 - 92,870$$

$$X = 602(1.06)^{26}$$

$$= 2,739$$

$$\Delta \text{Interest paid} = \Delta \text{Total Payments}$$

$$\text{Original payments} = 30(7264.89)$$

$$\text{New payments} = 25(7264.89) + 2(5,000) + 2,739$$

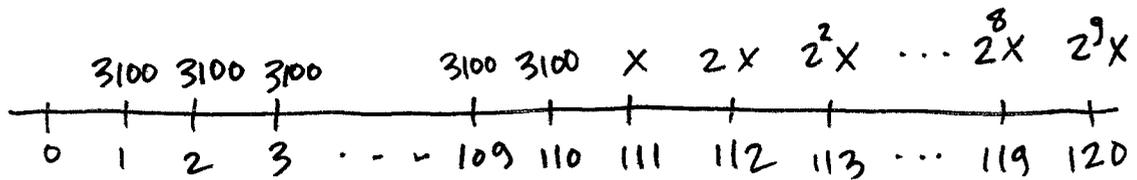
$$\Delta \text{ payments} = 5(7,264.89) - 10,000 - 2,739$$

$$= 23,585$$

(B)

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- 4 The key to this problem is carefully writing down all the payments on a time line diagram:



The final repayment is $512X$. You can solve for X by looking at time 0, or at time 110. After the 110th repayment, the remaining series of payments will fund the outstanding loan balance:

$$\begin{aligned} \text{OK Loan Balance} &= \text{Accumulated Loan} - \text{Accumulated payments} \\ &= 100,000(1.03)^{110} - 3100s_{\overline{10}|3\%} \\ &= 2,582,823 - 2,565,584 = 17,239 \end{aligned}$$

We are using 3% per quarterly compounding period, since the interest rate on a nominal basis is 12% per annum.

Now setup the PV of future payments to solve for X

$$\begin{aligned} 17,239 &= Xv + 2Xv^2 + 4Xv^3 + \dots + 512Xv^{10} \\ 2v(17,239) &= 2Xv^2 + 4Xv^3 + \dots + 512Xv^{10} + 1024Xv^{11} \end{aligned}$$

The trick is to see that when you multiply by $2v$, then subtract the equations, most of the terms cancel out

$$(2v-1)(17,239) = 1024Xv^{11} - Xv = X(1024v^{11} - v)$$

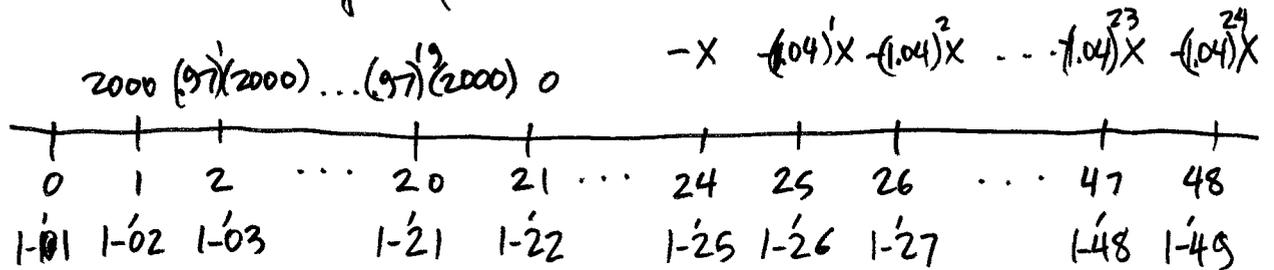
$$\begin{aligned} X &= \frac{(2v-1)(17,239)}{1024v^{11} - v} = \frac{.9417(17,239)}{1024(.7224) - .9709} \\ &= 16,235 / 738.79 \\ &= 21.97 \end{aligned}$$

$$\text{Final payment} = 512(21.97) = 11,251$$

(B)

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5 With the varying payments and withdrawals, the key to this problem is writing everything on a time line diagram:



You are told that the withdrawals eliminate the account at 1-1-49, which is after the 48th withdrawal. Be careful that you don't confuse the years and the payment numbers.

Next, calculate the accumulated value of the payments as of 1-1-49. Then you can set that equal to the accumulated value of the withdrawals, and solve for X:

$$\begin{aligned}
 \text{1-1-48 value} &= 2000 \left[(1.05)^{47} + .97(1.05)^{46} + \dots + (.97)^{19}(1.05)^{28} \right] \\
 \text{of payments} &= 2000(1.05)^{47} \left[1 + (.97/1.05) + \dots + (.97/1.05)^{19} \right] \\
 &= 2000(1.05)^{47} \ddot{a}_{\overline{20}|j} \text{ where } 1+j = \frac{1.05}{.97} = 1.0825 \\
 &= 2000(1.05)^{47} (1.0825) \ddot{a}_{\overline{20}|8.25\%}
 \end{aligned}$$

I have written this NOT using an annuity due intentionally. With an HP-12C calculator, I have set it to calculate immediate annuities. I do not change it between Due and Immediate, simply to avoid making a careless error.

$$\begin{aligned}
 \text{1-1-48 value} &= 2000(9.9060)(1.082474)(9.6400) \\
 &= 206,738
 \end{aligned}$$

Be sure to verify the correct # of payments (20) in the annuity above.
(next page)

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(5) The 1-48 value of the withdrawals is equal to 206,738:

1-48 value

$$\begin{aligned} \text{withdrawals} &= X(1.05)^{24} + (1.04)X(1.05)^{23} + \dots + (1.04)^{24}X \\ &= X(1.05)^{24} [1 + (1.04/1.05) + \dots + (1.04/1.05)^{24}] \\ &= X(1.05)^{24} \ddot{a}_{\overline{25}|k} \quad \text{where } 1+k = \frac{1.05}{1.04} = 1.0096 \end{aligned}$$

The number of withdrawals in the annuity (25) matches the data in the problem, as it should. I will evaluate the annuity using an Immediate, not a Due, as previously mentioned.

$$\begin{aligned} \text{1-48 value} &= X(1.05)^{24}(1+k)(\ddot{a}_{\overline{25}|k}) \\ \text{of withdrawals } 206,738 &= X(1.05)^{24}(1.0096)(22.1282) \\ X &= \frac{206,738}{72.0519} \\ &= 2,869 \end{aligned}$$

The question asked for the sum of withdrawals at $v/25$ and $v/26$:

$$X + 1.04X = 5,853$$



6 This problem is quite messy, and definitely feels like it is worth 5 points. As usual, you should carefully write down payments on a time line diagram.

With quarterly loan repayments, you must determine the quarterly interest rate. Interest is 4% per six months, based on the nominal 8% compounded semi-annually
 $(1.04)^{1/2} = 1.0198 \Rightarrow 1.98\%$ interest per quarter

The original loan had 32 payments = $25,000 / a_{\overline{32}|1.98\%}$.
 After the 10th payment, the o/s loan balance equals the present value of the remaining payments:

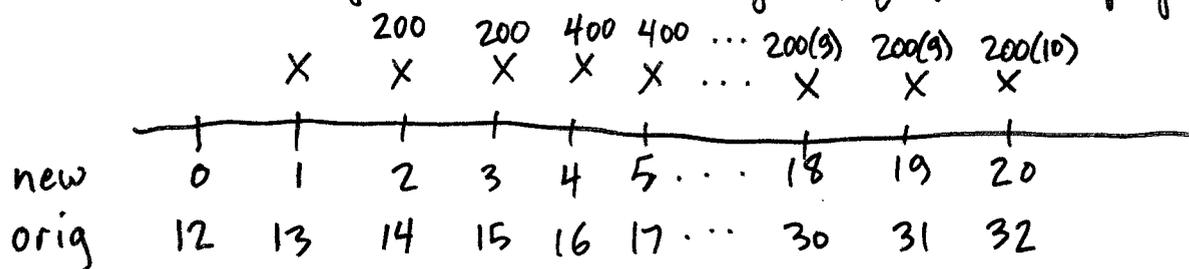
$$\text{o/s balance} = \left(\frac{25,000}{a_{\overline{32}|1.98\%}} \right) a_{\overline{22}|1.98\%}$$

After missing the 11th and 12th payments you have
 $\text{o/s balance} = (1.0198)^2 25,000 \left(\frac{a_{\overline{22}|}}{a_{\overline{32}|}} \right)$

$$= 1.04 (25,000) (17.6944 / 23.5354)$$

$$= 19,547$$

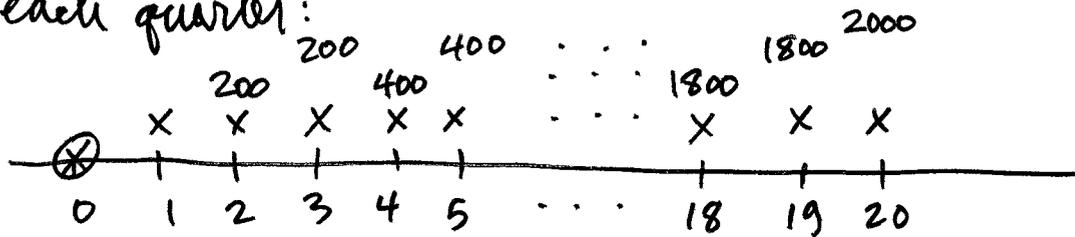
Now set up the time line diagram for the new payments



The value at the new time zero is 19,547.

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- (6) you have a level annuity of X , plus two increasing annuities. The tricky part is that you should value the increasing annuities based on the 4% semi-annual rate, since their payments are not made each quarter:



$$19,547 = X(a_{\overline{20}|1.98\%}) + 2000(1.0198)^{-20} + 200(Ia_{\overline{9}|.04}) \left[1 + \frac{1}{1.0198}\right]$$

Use the identity $Ia_{\overline{n}|i} = \frac{\ddot{a}_{\overline{n}|i} - nv^n}{i}$

$$\therefore Ia_{\overline{9}|.04} = \frac{1.04(a_{\overline{9}|.04}) - 9(1.04)^{-9}}{.04}$$

$$(a_{\overline{20}|1.98\%})X = 19,547 - 2000(1.0198)^{-20} - 200 \left[\frac{1.04(a_{\overline{9}|.04}) - 9(1.04)^{-9}}{.04} \right] \left(1 + \frac{1}{1.0198}\right)$$

$$X(16.3824) = 19,547 - 2000(.6756) - 200 \left[\frac{1.04(7.4353) - 9(.7026)}{.04} \right] 1.9806$$

$$X = \frac{19,547 - 1351.2 - 200(35.2366)1.9806}{16.3824}$$

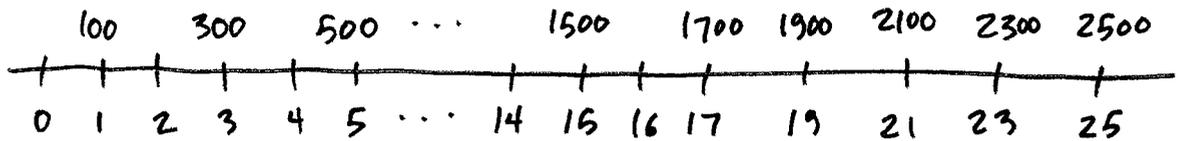
$$= 4238/16.3824$$

$$= 258.70$$

©

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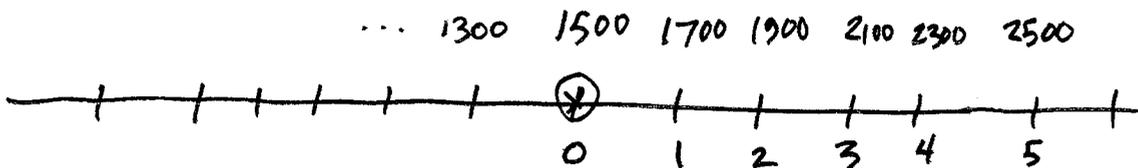
7 The key to this problem is writing down all the payments on a time line diagram



One simplification you can make is to use an interest rate that corresponds to the payment period. Since the payments are biennial, calculate the biennial interest:

$$(1.06)^2 - 1 = 12.36\% \text{ every 2 years}$$

Now rewrite the payments reflecting the biennial interest compounding period



The payments you are interested in are those made after the 15th year, which are the last 5 payments.

$$\begin{aligned} \text{You can easily calculate } A &= 1700 + 1900 + 2100 + 2300 + 2500 \\ &= 2(4000) + 2500 = 10,500 \end{aligned}$$

B is the present value at time zero - which corresponds to the beginning of the 16th year in the first diagram

$$\begin{aligned} B &= 1500 a_{\overline{5}|12.36\%} + 200 Ia_{\overline{5}|12.36\%} & Ia_{\overline{5}|} &= \frac{a_{\overline{5}|} - 5v^5}{i} \\ &= 1500(3.5729) + 200 \left[1.1236(a_{\overline{5}|}) - 5(1.1236)^5 \right] / 12.36 \\ B &= 7337 = 1500(3.5729) + 200(9.8907) \\ \therefore A - B &= 10,500 - 7,337 = 3,163 \quad \text{C} \end{aligned}$$

- 8 The key to this problem is knowing how to write down the amortization schedule for the loan. The original loan is $1000 a_{\overline{4}|i} = 1000 \frac{(1-v^4)}{i}$.

Now you can write the amortization schedule. The idea is that each payment pays off the interest due, which is i times last year's loan balance. The remainder of the payment pays off the principal.

Year	Payment	Interest	Principal	After payment o/s Loan (BOY)
1	1000	$1000(1-v^4)$	$1000v^4$	$1000 a_{\overline{3} i}$
2	↓	$1000(1-v^3)$	$1000v^3$	$1000 a_{\overline{2} i}$
3		$1000(1-v^2)$	$1000v^2$	$1000 a_{\overline{1} i}$
4		$1000(1-v)$	$1000v$	0

Now set up the relationship between principal and interest for the first 2 and last 2 years:

$$1000v^4 + 1000v^3 = 10v^2 [1000(1-v^2) + 1000(1-v)]$$

$$1000v^2(v^2+v) = 1000v^2 [10] [2-v-v^2]$$

$$v^2+v = 20 - 10v - 10v^2$$

$$11v^2 + 11v - 20 = 0$$

Now you use the quadratic formula to solve for v :

$$ax^2 + bx + c = 0 \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$v = \frac{-11 \pm \sqrt{(11)^2 - 4(11)(-20)}}{2(11)} = \frac{-11 \pm 31.6386}{22}$$

$$= -1.94 \text{ or } +.9381 \text{ (ignore negative) } \textcircled{C}$$

- 9) The key to this problem is knowing how to write down an amortization schedule for the loan. Another idea to simplify the solution is to convert the interest rate to match the payment period. Think of this as a loan with 10 payments, based on a triennial interest rate:
 $(1.04)^3 - 1 = 12.49\%$ every 3 years

<u>"Year"</u>	<u>Payment</u>	<u>Interest</u>	<u>Principal</u>	<u>After payment o/s Loan (AOY)</u>
0				$1000 = P \frac{(1-v^{10})}{i}$
1	P	$P(1-v^{10})$	Pv^{10}	$P \cdot a_{\overline{10} i}$
2		v^9	v^9	
3		v^8	v^8	
4		v^7	v^7	
5	P	$P(1-v^6)$	Pv^6	

The principal repaid in the fifth payment is Pv^6 .
 Solve for the value of P using the terms of the initial loan:

$$1000 = P a_{\overline{10}|12.49\%}$$

$$P = 180.52$$

$$Pv^6 = 180.52 (1.1249)^{-6} = 89.11$$

(B)

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- 10 You can't rely on the exam condition #9 to say that the redemption value is 10,000. That would be way too easy!

The key to this problem is knowing the relationship between the price of a bond at successive coupon dates. The amortized value is the price of the bond based on the unknown yield rate for the purchaser.

Each semi-annual coupon is $4\% (10,000) = 400$.

At 1-1-94 issue date there are 30 coupons to be paid

At 1-1-02, 8 years later, there are $30 - 16 = 14$ coupons left

$$\begin{aligned} 1-1-02 \text{ Amortized value} &= Fr a_{\overline{14}|i} + Cv^{14} \\ &= 400 a_{\overline{14}|i} + Cv^{14} \end{aligned}$$

The question asks for the redemption value C . You need to solve for the value of i based on the two values at 7-1-01 and 1-1-02:

$$7-1-01 \text{ Amortized value} = 400 a_{\overline{15}|i} + Cv^{15}$$

Remember the relationship $P_{t+1} = P_t(1+i) - Fr$

$$1-1-02 \text{ value} = (1+i)[7-1-01 \text{ value}] - 400$$

$$13,629.67 = (1+i)(13,741.11) - 400$$

$$1+i = 1.021$$

Now you can calculate the 1-1-02 amortized value and derive the value of the redemption amount C

$$13,629.67 = 400 a_{\overline{14}|2.1\%} + C(1.021)^{-14}$$

$$= 400(12.0215) + C(.7476) \Rightarrow C = 11,800$$

Ⓒ

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- 11 The key to working this problem is knowing how to calculate the amortized value of a bond. Instead of the prospective formula used in the prior problem, this time we'll use the retrospective version:

$$\text{Amortized value} = \text{Accumulated Price} - \text{Accumulated coupons}$$

Since the coupons are paid biennially, it makes sense to use a biennial interest rate that matches the coupons:
 $(1.08)^2 - 1 = 16.64\%$ every 2 years.

$$\begin{aligned} A = 1-1-05 \text{ Amortized value} &= 691.49(1.1664)^2 - 60 \overline{s}_{\overline{2}|16.64} \\ &= 810.78 \qquad \qquad \qquad = 691.49(1.3605) - 60(2.1664) \end{aligned}$$

$$\begin{aligned} B = 1-1-07 \text{ Amortized value} &= 691.49(1.1664)^3 - 60 \overline{s}_{\overline{3}|16.64} \\ &= 885.69 \qquad \qquad \qquad = 691.49(1.5869) - 60(3.5269) \end{aligned}$$

$$[A - B] = 74.91$$

(E) This is barely in the implied range of 72-75 for E

If you used the prospective formula, you would have one extra step in the problem to solve for the redemption value:

$$\begin{aligned} 1-1-01 \text{ Price} &= 691.49 = 60 \overline{a}_{\overline{5}|16.64\%} + FV^5 \\ 1-1-05 \quad A &= 60 \overline{a}_{\overline{3}|16.64\%} + FV^3 \\ 1-1-07 \quad B &= 60 \overline{a}_{\overline{2}|16.64\%} + FV^2 \end{aligned}$$

From the first formula, you can derive $F = 1075$. Then you can calculate the same values for A and B shown earlier.

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- 12 The key to this problem is counting the remaining number of payments carefully, and redetermining the outstanding loan balance and new monthly payment each time the interest rate changes.

Original Loan

200,000

$$\text{monthly rate} = 7\% / 12 = .5833\%$$

$$\begin{aligned} \text{New payment} &= 200,000 / a_{\overline{360}|.5833\%} \\ &= 1,330.60 \end{aligned}$$

7.5% Loan

After 24 payments

Remaining Loan Balance

$$1,330.60 a_{\overline{336}|.5833\%} = 195,790$$

$$\text{Monthly rate} = 7.5\% / 12 = .625\%$$

$$\begin{aligned} \text{New payment} &= 195,790 / a_{\overline{336}|.625\%} \\ &= 1,395.72 \end{aligned}$$

8% Loan

After 24 more payments

Remaining Loan Balance

$$1,395.72 a_{\overline{312}|.625\%} = 191,350$$

$$\text{Monthly rate} = 8\% / 12 = .6667\%$$

$$\begin{aligned} \text{New payment} &= 191,350 / a_{\overline{312}|.6667\%} \\ &= 1,459.23 \end{aligned}$$

The total interest paid is the difference between the total payments and the original loan amount.

$$\begin{aligned} \text{Total Payments} &= 24(1,330.60) + 24(1,395.72) + 312(1,459.23) \\ &= 31,935 + 33,497 + 455,281 = 520,713 \end{aligned}$$

$$\text{Total Interest} = 320,713 = 520,713 - 200,000 \text{ Loan } \textcircled{D}$$

- 13 The key to this problem is knowing how to calculate the present value of a perpetuity. You also need to know the definition of the modified duration.

$$\text{PV of perpetuity} = (1.06)^{-1} + (1.06)^{-2} + \dots = \frac{1}{1.06}$$

If you had a perpetuity due, the PV is $1/d$, not $1/i$.

The definition of modified duration is that it equals the Macaulay duration divided by $1+i$. The Macaulay duration equals the ratio of the weighted average future period to each payment, divided by the present value. The weight used in the numerator is the present value of each payment:

$$\bar{d} = \frac{\sum_{t=1}^n t v^t R_t}{\sum_{t=1}^n v^t R_t} \quad (R_t \text{ is series of pmts})$$

$$= (1 \cdot (1.06)^{-1} + 2 \cdot (1.06)^{-2} + \dots) / (\text{Present value})$$

The numerator is an increasing perpetuity, which has the general formula $1/i + 1/i^2$. Now you can determine the generalized result for both the regular (Macaulay) duration, and the modified duration:

$$\bar{d} = \frac{1/i + 1/i^2}{1/i} = 1 + 1/i = \frac{1+i}{i}$$

$$\text{modified duration} = \bar{d} / (1+i) = \frac{(1+i/i)}{(1+i)} = 1/i$$

\therefore modified duration = PV for perpetuity immediate, $\Delta = \text{zero}$ (A)

- 14 This is an extremely long problem, as it usually is for 5 points. The key to the problem is interpretation of the wording. You are told that X and Y are "the amount invested." This is not the face amount of the bonds, but rather it is the price paid, which equals the present value of each bond.

You can calculate the present value and the modified duration for Portfolio C. Then you can use those values to determine the bonds in the other two portfolios.

Portfolio C

$$PV = 10,000v^4 = 7628.95$$

$$\text{Macaulay Duration} = 4 = \frac{4(10,000)v^4}{(10,000)v^4}$$

$$\text{modified Duration} = \frac{4}{1.07}$$

For simplicity, you can use the Macaulay duration of 4 for each portfolio, since the modified durations will all equal $4/1.07$.

Portfolio A

You are told that X is the amount invested (present value) of the zero coupon bonds. Let W be the face amount of the four year bonds. You need to set up expressions for the present value, and the regular (Macaulay) duration. By setting these equal to the values for Portfolio C, you have 2 equations

(14) with 2 unknowns.

$$PV = W [i a_{\overline{4}|i} + v^4] + X \quad i = 7\%, \text{ also coupon rate}$$

For the zero coupon bond, let the face amount equal Q , for purposes of writing a formula for the regular duration:

$$\text{Regular Duration} = \frac{W [i (1v + 2v^2 + 3v^3 + 4v^4) + 4v^4] + 5Qv^5}{W [i a_{\overline{4}|i} + v^4] + Qv^5} = \bar{d}$$

Note $X = Qv^5$, so now state in terms of X

$$\bar{d} = \frac{.07W [I a_{\overline{4}|.07}] + W(4v^4) + 5X}{W + X}$$

The denominator is the present value of the two bonds. Since the coupon rate and yield rate are both 7%, the present value of any n year bond will equal the face. remember $I a_{\overline{n}|i} = \frac{\ddot{a}_{\overline{n}|i} - nv^n}{i} = \frac{(1+i) a_{\overline{n}|i} - nv^n}{i}$

As mentioned in solution #5, I always use immediate annuities with an HP-12C calculator.

$$\begin{aligned} \bar{d} &= \frac{.07W [(1.07) a_{\overline{4}|.07} - 4v^4] + W4v^4 + 5X}{W + X} \\ &= \frac{W(1.07) a_{\overline{4}|.07} + 5X}{W + X} \end{aligned}$$

Now use the PV and duration for Portfolio C:

$$\bar{d} = 4 = \frac{W(1.07) a_{\overline{4}|.07} + 5X}{7628.95}$$

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(14) You should use substitution for W (based on the present value) to solve for X :

$$PV = 7628.95 = W + X$$

$$W = 7628.95 - X$$

$$W(1.07)^4 + 5X = 4(7628.95)$$

$$(7628.95 - X)(1.07)(3.3872) + 5X = 4(7628.95)$$

$$7628.95(3.6243) - 3.6243X + 5X = 4(7628.95)$$

$$1.3757X = .3757(7628.95)$$

$$X = 2083.38$$

Now, apply essentially the same steps to Portfolio B. This time, let the face amount of the 3 year bond be Z . For the zero coupon bond, let its face be R :

$$PV = Z[ia\bar{a}_{\bar{3}|i} + v^3] + Rv^5$$

$$= Z + Y = 7628.95$$

$$\text{Regular Duration } \bar{d} = \frac{Z[i(1 \cdot v + 2 \cdot v^2 + 3 \cdot v^3) + 3v^3] + 5Rv^5}{Z + Y}$$

$$= \frac{.07Z(1A\bar{a}_{\bar{3}|.07}) + Z3v^3 + 5Y}{Z + Y}$$

$$= \frac{.07Z[(1.07(a\bar{a}_{\bar{3}|.07}) - 3v^3)/.07] + Z3v^3 + 5Y}{Z + Y}$$

$$\text{Finally } 4 = (Z(1.07)a\bar{a}_{\bar{3}|.07} + 5Y) / 7628.95$$

Solve for the value of Y by using a similar substitution based on the present value:

$$Z = 7628.95 - Y$$

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(14)

$$\begin{aligned}Z(1.07) 237.07 + 5Y &= 4(7628.95) \\(7628.95 - Y)(1.07)(2.6243) + 5Y &= 4(7628.95) \\7628.95(2.8080) - 2.8080Y + 5Y &= 4(7628.95) \\2.1920Y &= 1.1920(7628.95) \\Y &= 4148.56\end{aligned}$$

$$Y - X = 2065.18 \quad \textcircled{B}$$

In hindsight, the best choice is to skip this problem on the test. I don't think you can get 5 points without using up too much time working this one!

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- 15 The key to working this problem is writing out the series of payments, and expressing it as an annuity. In effect, the net interest rate is based on the ratio of 1.08 to $(1 + \text{CPI rate})$.

One minor point is interpretation of the indexing of the payments. Each year's payment should be increased by multiplying by $(1+j)^n$ where $1+j = 1 + \text{CPI rate} - 3\%$.

$$\begin{aligned}
 X &= \frac{10,000}{1.08} + \frac{10,000(1.03)^1}{(1.08)^2} + \dots + \frac{10,000(1.03)^{11}}{(1.08)^{12}} \\
 &= \frac{10,000}{1.08} \left[1 + \frac{1.03}{1.08} + \dots + \left(\frac{1.03}{1.08}\right)^{11} \right] \\
 &= (10,000/1.08) \ddot{a}_{\overline{12}|k} \text{ where } 1+k = 1.08/1.03 = 1.0485 \\
 &= 9,259 (1.0485) \ddot{a}_{\overline{12}|4.85\%} \\
 &= 86,762
 \end{aligned}$$

$$\begin{aligned}
 Y &= \frac{10,000}{1.08} + \frac{10,000(1.01)^1}{(1.08)^2} + \dots + \frac{10,000(1.01)^{11}}{(1.08)^{12}} \\
 &= (10,000/1.08) \ddot{a}_{\overline{12}|m} \text{ where } 1+m = 1.08/1.01 = 1.0693 \\
 &= 9,259 (1.0693) \ddot{a}_{\overline{12}|6.93\%} \\
 &= 78,932
 \end{aligned}$$

$$[X - Y] = 7,830.39$$

(B)

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16 The key to this problem is realizing that the mortality corresponds to DeMoivre's law:

$$l_0 = 100 \quad d_x = 1 \quad l_{100} = 0$$

If you know how to calculate the annuity under deMoivre's law, the problem is not very long:

$$a_x = \frac{n - \ddot{a}_{\overline{n}|i}}{n(i)} \quad \text{where } n = w - x$$

$$x = 60 \quad w = 100 \quad n = 40$$

$$\begin{aligned} a_{60} &= \frac{40 - \ddot{a}_{\overline{40}|.06}}{40(.06)} \\ &= \frac{40 - 1.06(a_{\overline{40}|.06})}{2.4} \\ &= \frac{40 - 15.9491}{2.4} \\ &= 10.0212 \end{aligned}$$

$$\therefore 100 a_{60} = 1002.12$$

(B)

If you have to derive the formulas from scratch, then the amount of work required makes this seem like a 4 point problem!

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- 17 The key to this problem is carefully identifying which ages' payments are contingent on survival, and which are not. In general terms the annuity present value is as follows:

$$\begin{aligned} PV &= v p_{105} (a_{\overline{106:\overline{2}|}}) 1000 \\ &= v p_{105} (v + v^2 + v^3 p_{106} + v^4 p_{106} + \dots) 1000 \end{aligned}$$

Based on the formula given for l_x , there are no payments after age 109. The value of l_{100} = zero:

<u>Age x</u>	<u>l_x</u>
105	950
106	760 = .8(950)
107	570 = .6(950)
108	380 = .4(950)
109	190 = .2(950)
110	0

$$\begin{aligned} PV &= v (760/950) [v + v^2 + v^3 (190/760)] 1000 \\ &= v (.80) [v + v^2 + v^3 (.25)] 1000 \\ &= (.80/1.07) [.9346 + .8734 + .8163(.25)] 1000 \\ &= 1504.36 \end{aligned}$$

(B)

Since you were calculating p_x values, you could have skipped the actual calculation of the values of l_x . The p_x values come directly from ratios of the formula.

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- 18 This problem has appeared on the exam many times: 1992 #3, 1995 #21, and 2000 #2. See 1992 #3 for the more difficult approach that can be used.

The value of the insurance is the difference between the mortgage of 50,000, and the value of mortgage payments that would be paid while the mortgagee is still alive.

$$\begin{aligned} \text{Mortgage payment} &= 50,000 / a_{\overline{31}|.05} \\ \text{PV of Insurance} &= 50,000 - a_{\overline{57}|.31} \left(\frac{50,000}{a_{\overline{31}|.05}} \right) \end{aligned}$$

The majority of the work is simply calculating the annuity values. In the 1992 problem, you are given commutation functions, which simplified the calculations.

$$\begin{aligned} a_{\overline{57}|.31} &= v^1 p_{57} + v^2 {}_2p_{57} + v^3 {}_3p_{57} \\ &= \frac{.858/1.05 + .859/(1.05)^2 + .860/(1.05)^3}{.857} \\ &= \frac{.8141/1.05 + .8505/1.1025 + .8467/1.1576}{.8574} \\ &= (9087 + 8621 + 8178) / 9574 \\ &= 2.7038 \end{aligned}$$

$$a_{\overline{31}|.05} = 2.7232$$

PV of

$$\begin{aligned} \text{Insurance} &= 50,000 (1 - a_{\overline{57}|.31} / a_{\overline{31}|.05}) \\ &= 50,000 (1 - 2.7038 / 2.7232) \\ &= 357.55 \end{aligned}$$

(B)

- 19) The key to this problem is knowing the definition of a certain and life annuity. Since you have no commutation functions, you must express everything in simplest terms to calculate the numerical answer.

$$\text{Original PV} = 50,000 \ddot{a}_{65} (D_{65}/D_{55})$$

$$\text{New PV} = 50,000 (\ddot{a}_{70} + {}_{10|}\ddot{a}_{65}) D_{65}/D_{55}$$

$$\Delta \text{PV} = 50,000 (\ddot{a}_{70} - \ddot{a}_{65} : {}_{10|}) v^{10} {}_{10}p_{55}$$

$$\ddot{a}_{70} = 1.06 (a_{70|.06}) = 7.8017$$

$$\begin{aligned} \ddot{a}_{65} : {}_{10|} &= \ddot{a}_{65} - v^{10} {}_{10}p_{65} (\ddot{a}_{75}) \\ &= (1 + a_{65}) - (1.06)^{-10} \frac{l_{75}}{l_{65}} (1 + a_{75}) \end{aligned}$$

You need to calculate l_{75}/l_{65} . Since ${}_{10}p_{55} = l_{65}/l_{55}$ and ${}_{20}p_{55} = l_{75}/l_{55}$, $l_{75}/l_{65} = {}_{20}p_{55}/{}_{10}p_{55} = .624/.920 = .6783$

$$\begin{aligned} \ddot{a}_{65} : {}_{10|} &= 9.897 - .5584 (.6783) (7.217) \\ &= 7.1637 \end{aligned}$$

$$\begin{aligned} \Delta \text{PV} &= 50,000 (7.8017 - 7.1637) (.5584) (.920) \\ &= 16,388.79 \quad \text{(A)} \end{aligned}$$

20 The key to this problem is knowing how to calculate both e_x and \dot{e}_x . In addition, you need to know that you won't use the information re: ${}_t p_{51}$.

e_x can be thought of as similar to a_x , but with a zero interest rate. \dot{e}_x is the complete expectation of life, which includes an allowance for the year of death.

$$e_x = p_x + 2p_x + 3p_x + \dots$$

$$\dot{e}_x \doteq e_x + \frac{1}{2}$$

$$e_{x+1} = p_{x+1} + 2p_{x+1} + 3p_{x+1} + \dots$$

$$e_x = p_x [1 + p_{x+1} + 2p_{x+1} + \dots]$$

$$= p_x [1 + e_{x+1}]$$

$$e_{x+1} = p_{x+1} [1 + e_{x+2}]$$

Once you develop the relationship for the expectation of life at consecutive ages, you are ready to calculate the value:

$$\dot{e}_{50} \doteq e_{50} + \frac{1}{2}$$

$$\dot{e}_{52} \doteq e_{52} + \frac{1}{2}$$

$$e_{52} \doteq 26.0 - .5 \\ = 25.5$$

$$e_{50} = p_{50} [1 + e_{51}]$$

$$e_{51} = p_{51} [1 + e_{52}]$$

$$= \frac{l_{52}}{l_{51}} [1 + 25.5]$$

$$= 25.8375$$

$$e_{50} = \frac{l_{51}}{l_{50}} [1 + 25.8375]$$

$$= 26.3007$$

$$\dot{e}_{50} \doteq e_{50} + \frac{1}{2} = 26.80$$

Ⓒ

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- 21 This problem is similar to 2000 #19. The key is knowing the formula for the average age at death. In addition, you need to know how to calculate e_x based on data from the stationary population.

T_x represents the number of years lived from age x until death by the l_x individuals attaining age x in a given year. It should make sense that $e_x = \frac{T_x}{l_x}$.

Based on the data given, you need to derive the value of l_{50} . You are given the value of T_{50} as 2700. It is typical that T_x has another interpretation, which is the number of people currently at age x and older in the stationary population.

$$\text{Average age at death between ages } x \text{ and } y = x + \frac{T_x - T_y - (y-x)l_y}{l_x - l_y}$$

$$33\frac{1}{3} = 20 + \frac{T_{20} - T_{50} - 30l_{50}}{l_{20} - l_{50}}$$

$$13\frac{1}{3}(l_{20} - l_{50}) = 21,600 - 2,700 - 30l_{50}$$

$$16\frac{2}{3}l_{50} = 21,600 - 2,700 - 13\frac{1}{3}(1080)$$

$$l_{50} = 4,500 / 16.6667$$

$$= 270.00$$

$$\begin{aligned} \text{Finally you have } e_{50} &= T_{50} / l_{50} \\ &= 2700 / 270 \\ &= 10.0 \end{aligned}$$

(C)

- 22 This is a fairly typical exam question. The key is knowing how to determine the probability that matches the events described in the problem. In addition, you need to know how to use the relationship between successive e_x values to get the probability of survival.

$$a_x = v p_x (1 + a_{x+1})$$

$$e_x = p_x (1 + e_{x+1})$$

$$p_x = e_x / [1 + e_{x+1}]$$

$$p_{40} = 16.5/17.2 \\ = .9593$$

$$p_{41} = 16.2/17.0 \\ = .9529$$

$$p_{42} = 16.0/16.8 \\ = .9524$$

Next, write down all possible cases for death or survival during 2001. Then identify which ones will meet the statement of the question, based on experience in 2002:

<u>2001</u>	<u>2002</u>	<u>probability</u>
Both alive	Don't care	—
Both dead	Don't care	—
Smith ONLY alive	Smith dies	$(p_{40} q_{41}) q_{41}$
Brown ONLY alive	Brown dies	$(q_{40} p_{41}) q_{42}$

It is easy to get confused by looking at the p 's and q 's, since the ages change from one year to the next. Finally you should plug in the p_x values from above:

$$\begin{aligned} & p_{40} (q_{41}) q_{41} + q_{40} (p_{41}) q_{42} \\ & .9593 (1 - .9529)^2 + (1 - .9593)(.9529)(1 - .9524) \\ & = .00212 + .00195 \\ & = .00397 \end{aligned}$$

(C)

- 23 This is a probability question that often appears on the exam. One unusual aspect is that the probabilities appear to be based on different mortality tables for each participant.

The key to the question is that you should not just write down the expressions based on p_x . You need to carefully write down the values for various cases.

All 3 alive after 15 years	$(.95)^{15}(.90)^{15}(.85)^{15}$
Smith + Brown alive, Green dead	$(.95)^{15}(.90)^{15}[(.85)^5 - (.85)^{15}]$
Smith + Green alive, Brown dead	$(.95)^{15}[(.90)^5 - (.90)^{15}](.85)^{15}$
Smith death, Green + Brown alive	$[(.95)^5 - (.95)^{15}](.90)^{15}(.85)^{15}$

The terms in square brackets represent the probability of surviving for 5 years, but dying before the end of 15 years. Now you can add and combine terms, and finally calculate the probability:

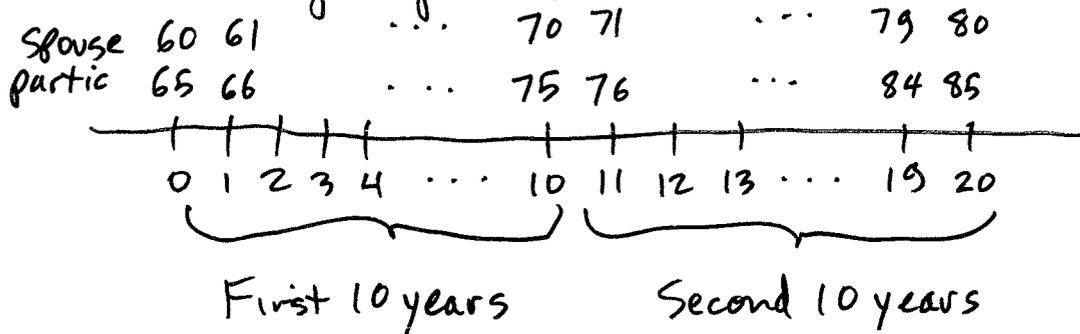
$$\begin{aligned}
 & (.95)^{15}(.90)^{15}(.85)^{15} & = & .04232 \\
 & - 2(.95)^{15}(.90)^{15}(.85)^{15} & = & -.01667 \\
 & + (.95)^{15}(.90)^5(.85)^{15} & = & .02390 \\
 & + (.95)^5(.90)^{15}(.85)^{15} & = & .01392 \\
 & & & \hline
 & & & .0635
 \end{aligned}$$

(B)

Storing the different values in six registers in the calculator should make the final calculation easier.

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24 This is a difficult thought problem. The key is to understand how to set up the probabilities, based on the information you are given. First, write down the ages of the participant and spouse for the entire 20 year period:



Even if the spouse died immediately, the participant would receive payments for the first 10 years, as long as the participant is alive. Once you realize this is true, it is easier to construct the expression for the last 10 years of payments:

$$v^{10} {}_{10}p_{65} [v^1 p_{75} p_{60} + v^2 {}_2p_{75} {}_2p_{60} + \dots + v^{10} {}_{10}p_{75} {}_{10}p_{60}]$$

For the participant to receive the 11th payment, the spouse must live at least one year, and the participant must survive to age 76 to receive the payment. The expression in brackets is $a_{75:60:\overline{10}|}$, a temporary immediate annuity. For the first ten years, you use $a_{65:\overline{10}|}$:

$$\begin{aligned}
 PV &= 10,000 [a_{65:\overline{10}|} + v^{10} {}_{10}p_{65} (a_{75:60:\overline{10}|})] \\
 &= 10,000 [7.72174 + .54544 (6.49715)] = 112,655 \text{ (D)}
 \end{aligned}$$

25 This is a messy probability question. You must write out each of the items that you are given, and then try to see how to derive $25P_{30}$. The confusing aspect is that there are several ways to write each probability based on combining terms differently. The real key is whether you see how to collapse things to a reducible form.

initial version

$$(i) 10P_{30}(10P_{40})(10P_{50}) = .758$$

$$(ii) (1-5P_{55})(5P_{50}) = .063$$

$$(iii) 5P_{30}(5P_{35})(5P_{40})(5P_{45})[1-5P_{50}] = .045$$

Since item (ii) is based on probabilities at ages 55 and 50, you want to express (i) and (iii) in a way that has similar terms:

$$(i) 20P_{30} = .758 = 25P_{30}(5P_{55}) = 20P_{30}(10P_{50})$$

$$(ii) 5P_{50} - 10P_{50} = .063$$

$$(iii) 20P_{30} - 25P_{30} = .045 = 20P_{30}(1-5P_{50})$$

I rewrote both (i) and (iii) so they had a term similar to item (ii). Now you have 3 equations in 3 unknowns and you can solve for the values of $20P_{30}$, $5P_{50}$ and $10P_{50}$. The problem asks for $25P_{30}$, which you can derive from (i) after you have calculated the value of $5P_{55}$:

$$5P_{55} : 25P_{30} = \frac{.758}{5P_{55}} ; 5P_{55} = 1 - \frac{.063}{5P_{50}}$$

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(25) Next, divide (i) by (ii) to eliminate $z\beta_{30}$:

$$(iv) = \frac{(i)}{(ii)} = \frac{.758}{.045} = \frac{10\beta_{50}}{1-s\beta_{50}} = 16.8444$$

Now rewrite (iv) using $s\beta_{55}$

$$(iv) \quad \frac{s\beta_{50}(s\beta_{55})}{1-s\beta_{50}} = 16.8444$$

$$s\beta_{50}(s\beta_{55}) = 16.8444 - 16.8444(s\beta_{50})$$

Now substitute for $s\beta_{50}$ based on the original version of (ii)

$$s\beta_{50} = \frac{.063}{1-s\beta_{55}}$$

The resulting equation allows you to solve for $s\beta_{55}$ directly

$$\frac{.063}{1-s\beta_{55}} (s\beta_{55}) = 16.8444 - 16.8444 \left(\frac{.063}{1-s\beta_{55}} \right)$$

$$.063 (s\beta_{55}) = 16.8444(1-s\beta_{55}) - 16.8444(.063)$$

$$16.9074 (s\beta_{55}) = 16.8444(1-.063)$$

$$s\beta_{55} = .9335$$

Now you can calculate $z\beta_{30} = \frac{.758}{s\beta_{55}}$

$$= .812$$

(B)

- 26 This is the second time this particular multiple decrement problem was asked on the exam. Most earlier problems assume uniform distribution of decrements in the multiple decrement tables. This problem assumes uniform distribution of decrements in the single decrement tables.

See problem 11 on the 2000 exam for the derivation of this formula, plus another example for this unusual question. Based on U.D.D. in the single decrement tables, we have $q_x^{(1)} \doteq q_x^{(1)} [1 - \frac{1}{2} q_x^{(2)}]$ which can be rewritten as $q_x^{(1)} \doteq \frac{q_x^{(1)}}{1 - \frac{1}{2} q_x^{(2)}}$

Based on the information given, you have $l_{50}^{(T)} = 100$, and $q_{50}^{(d)} = 16/100 = .16$.

$$\begin{aligned} &= q_{50}^{(d)} [1 - \frac{1}{2} q_{50}^{(w)}] \\ .16 &= q_{50}^{(d)} [1 - .5(.40)] \\ \therefore q_{50}^{(d)} &= .20 \end{aligned}$$

$$\begin{aligned} q_{50}^{(w)} &= q_{50}^{(w)} [1 - \frac{1}{2} q_{50}^{(d)}] \\ &= .4 [1 - .5(.20)] \\ &= .36 \end{aligned}$$

(A)

(Next Page)

- (26) The key point to remember is the relationship between the formulas based on U.D.D. in the single decrement table, and the formulas based on U.D.D. in the multiple decrement tables:

Single decrement UDD $q'_x^{(1)} \doteq \frac{q_x^{(1)}}{1 - \frac{1}{2} q'_x^{(2)}}$

Multiple decrement UDD $q'_x^{(1)} \doteq \frac{q_x^{(1)}}{1 - \frac{1}{2} q_x^{(2)}}$ Jordan 14.31b

One formula uses the absolute rate of decrement in the denominator, and the other uses the probability of decrement.

27 This is a typical multiple decrement problem, which has been asked many times on the exam. The key is knowing the various formulas that relate the single decrement table rates and the multiple decrement probabilities.

Jordan 1435: $p'_x^{(2)} \doteq [p_x^{(T)}]^{q_x^{(2)}/q_x^{(T)}}$

This formula works for ANY number of decrements, and it assumes uniform distribution of decrements.

$$p_x^{(T)} = p_x^{(1)} \cdot p_x^{(2)} = .90(.70) = .63$$

$$q_x^{(T)} = 1 - p_x^{(T)} = .37$$

$$.70 \doteq (.63)^{q_x^{(2)}/.37}$$

$$\log(.70) = \frac{q_x^{(2)}}{.37} (\log(.63))$$

$$q_x^{(2)} = .37 \frac{\log(.70)}{\log(.63)}$$

$$= .28563$$

(E)

It is disturbing how close this is to the end of the answer range! In addition, there is something fishy about the data in the problem. There are only two significant digits in the l_x values. But the answer ranges have 5 significant digits, which is inconsistent.

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(27) To illustrate the effect of the "imprecise" l_x values, see what the answer is based on other standard approximations

$$\text{Jordan 14.31b} \quad q_x^{1(2)} \doteq \frac{q_x^{(2)}}{1 - \frac{1}{2} q_x^{(1)}} = \frac{q_x^{(2)}}{1 - \frac{1}{2} (q_x^{(1)} - q_x^{(2)})}$$

This formula is based on uniform distribution of decrements. The first form assumes 2 decrements, but the second form is valid for any number of decrements.

$$q_{40}^{1(2)} \doteq \frac{q_{40}^{(2)}}{1 - \frac{1}{2} (q_{40}^{(1)} - q_{40}^{(2)})}$$

$$q_{40}^{(1)} = 1 - .70 = .30$$

$$q_{40}^{(2)} = 1 - p_{40}^{(2)} = 1 - .70(.90) = .37$$

$$.30 \doteq \frac{q_{40}^{(2)}}{1 - .5 (q_{40}^{(1)} - q_{40}^{(2)})}$$

$$.30 - .15 [q_{40}^{(1)} - q_{40}^{(2)}] = q_{40}^{(2)}$$

$$.30 - .15(.37) = .85 q_{40}^{(2)}$$

$$q_{40}^{(2)} = .2445 / .85$$

$$= .28765$$

Relatively speaking, this is a much larger result than previously calculated. Usually the answers don't vary much depending on which approximation is used.

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(27) Just for completeness, there is one more formula to try:

$$\text{Jordan 14.38} \quad q_x^{(2)} \doteq \frac{q_x^{(2)} [1 - .5 q_x^{(1)}]}{1 - .25 q_x^{(1)} q_x^{(2)}}$$

This formula is unique in that you can calculate the probability using only the single decrement rates. The formula is based on uniform distribution of decrements, and it is only valid for 2 decrements.

$$\begin{aligned} q_{40}^{(2)} &\doteq \frac{q_{40}^{(2)} [1 - .5(q_{40}^{(1)})]}{1 - .25 q_{40}^{(1)} q_{40}^{(2)}} \\ &= \frac{.30 (1 - .5(.10))}{1 - .25(.10)(.30)} \\ &= \frac{.285}{.9925} \\ &= .28715 \end{aligned}$$

This result is much closer to what resulted from the second formula, as opposed to the first result. The good news is that all of these answers fall in range E.

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28 This is a typical actuarial equivalence problem, but more difficult than many past problems. With actuarially equivalent benefits, the present values of all the options are equal. One key is being able to write the correct expressions for the present values.

But after that is done, you typically have "N" equations with "N+1" unknowns. The real key is finding the algebraic manipulation to get the same number of unknowns as there are equations!

Option I $12(4000)\ddot{a}_x^{(12)}$ assuming age x for pensioner

Option II $12(4000)\ddot{a}_x^{(12)} = 3600(12) [\ddot{a}_x^{(12)} + .5(\ddot{a}_y^{(12)} - \ddot{a}_{xy}^{(12)})]$

Option III $12(4000)\ddot{a}_x^{(12)} = 3582(12)\ddot{a}_{xy}^{(12)} + 1791(12)(\ddot{a}_y^{(12)} - \ddot{a}_{xy}^{(12)}) + 4000(12)(\ddot{a}_x^{(12)} - \ddot{a}_{xy}^{(12)})$

Option IV $12(4000)\ddot{a}_x^{(12)} = 12K\ddot{a}_{xy}^{(12)} + 6K(\ddot{a}_x^{(12)} - \ddot{a}_{xy}^{(12)}) + 6K(\ddot{a}_y^{(12)} - \ddot{a}_{xy}^{(12)})$
 $= 6K(\ddot{a}_x^{(12)}) + 6K(\ddot{a}_y^{(12)})$

Based on the equations for options II, III and IV you now have 3 equations, but 4 unknowns. To get rid of one, divide both sides of each equation by $12\ddot{a}_x^{(12)}$.

(next page)

- (28) There is one thing you may want to do first, which is to get rid of all the annuity functions. Since you have no annuity values, the loss of the relationships between the annuities won't affect the solution. At this point, it is only an algebra problem.

Replace $\ddot{a}_x^{(12)}$ with X , $\ddot{a}_y^{(12)}$ with Y , $\ddot{a}_{xy}^{(12)}$ with W

$$\text{II: } 12(4000X) = 3600(12)[X + .5(Y - W)]$$

$$\text{III: } 12(4000X) = 3582(12)[W] + 1791(12)(Y - W) + 4000(12)(X - W)$$

$$\text{IV: } 12(4000X) = 6K[X + Y]$$

It certainly is faster to write down X, Y, W instead of the annuities. Now divide both sides by $12X$

$$\text{II: } 4000 = 3600 [1 + .5(Y/X) - .5(W/X)]$$

$$\text{III: } 4000 = 3582 [W/X] + 1791(Y/X - W/X) + 4000(1 - W/X)$$

$$\text{IV: } 4000 = .5K [1 + Y/X]$$

You can simplify again—replace Y/X with A , and W/X with B .

$$\text{II: } 4000 = 3600 (1 + .5A - .5B)$$

$$\text{III: } 4000 = 3582B + 1791(A - B) + 4000 - 4000B$$

$$0 = 1791A - 2209B$$

$$\text{IV: } 4000 = .5K (1 + A)$$

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(28) At this point, the solution should be clear. Substitute the value of B from equation III into II. Then you have 2 equations (II and III) in 2 unknowns (A and K).

$$\text{II: } 4000 = 3600 \left(1 + .5A - .5 \left(\frac{1791A}{2209} \right) \right)$$

$$1.1111 = 1 + .5(.1892A)$$

$$A = \frac{.1111}{.0946} = 1.1744$$

$$\text{IV: } 4000 = .5K(1 + 1.1744)$$

$$.5K = 1839.6$$

$$K = 3679$$

(B)

You can try working this with the annuities written out, but I simply found the problem too confusing!

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29 This is a typical actuarial equivalence problem, and far more manageable than the previous problem. The key is being able to write down the correct expression for the annuities:

$$12(500) \ddot{a}_{\overline{65:\overline{5}}|}^{(12)} = 12X \ddot{a}_{65}^{(12)} + 6X (\ddot{a}_{65}^{(12)} - \ddot{a}_{65:65}^{(12)})$$

$$X = \frac{6000 (\ddot{a}_{57}^{(12)} + v^5 {}_5p_{65} \ddot{a}_{70}^{(12)})}{18 \ddot{a}_{65}^{(12)} - 6 \ddot{a}_{65:65}^{(12)}}$$

The only thing you are not given is $\ddot{a}_{57.05}^{(12)}$. I'll calculate the monthly interest rate, then evaluate $\ddot{a}_{60|j}$:

$$\begin{aligned} \ddot{a}_{60|j} : \quad & i = .05 \\ \text{monthly } j &= [(1.05)^{1/12} - 1] 12 \\ &= .4047\% \end{aligned}$$

$$\begin{aligned} \ddot{a}_{57.05}^{(12)} &= \frac{\ddot{a}_{60|.4074\%}}{12} \\ &= \frac{1.004074 (53.1338)}{12} = 4.4459 \end{aligned}$$

$$\begin{aligned} X &= \frac{6000 (4.4459 + (1.05)^{-5} (.95609)(11.27))}{18(12.80) - 6(10.87)} \\ &= \frac{6000 (4.4459 + 8.4426)}{165.18} \\ &= 468.16 \quad \textcircled{C} \end{aligned}$$