



Software Polish

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# SPRING 1990 EA-1A EXAM SOLUTIONS

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Revision History:

02/23/99    Enhanced problem 2    added faster method of solution

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1.  $S(x) = \frac{c-x}{c+x}$  = probability of survival from age 0 to age x

$$\frac{l_{35}}{l_0} = .44 = S(35) = \frac{c-35}{c+35} \quad .44c + 15.4 = c - 35$$

$$60.4 = .56c$$

$$c = 90$$

probability that Smith and Brown both survive to age 45 =  $({}_5p_{40})({}_5p_{40})$

probability that at least one survives from 45 to 60  
 = 1 - probability both die between 45 and 60 =  $1 - ({}_{15}q_{45})({}_{15}q_{45})$

$$X = ({}_5p_{40})({}_5p_{40}) \left[ 1 - ({}_{15}q_{45})({}_{15}q_{45}) \right]$$

$$= \left( \frac{l_{45}}{l_{40}} \right)^2 \left[ 1 - \left( \frac{l_{45} - l_{60}}{l_{45}} \right)^2 \right]$$

$$= \left( \frac{l_{45}/l_0}{l_{40}/l_0} \right)^2 \left[ 1 - \left( \frac{l_{45}/l_0 - l_{60}/l_0}{l_{45}/l_0} \right)^2 \right]$$

$$= \left[ \frac{S(45)}{S(40)} \right]^2 \left[ 1 - \left( 1 - \frac{S(60)}{S(45)} \right)^2 \right]$$

$$= \left[ \frac{(90-45)/(90+45)}{(90-40)/(90+40)} \right]^2 \left[ 1 - \left\{ 1 - \frac{(90-60)/(90+60)}{(90-45)/(90+45)} \right\}^2 \right]$$

$$= \left( \frac{.33\bar{3}}{.3846} \right)^2 \left[ 1 - \left\{ 1 - \frac{.20}{.333} \right\}^2 \right]$$

$$= (.8667)^2 [1 - (.4)^2]$$

$$= .6309$$

(C)

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Added solution 2/23/99

2 One way to work this problem uses Makeham's formula for the price of a bond:

$$P = K + (g/i)(C-K) \\ = Cv^n + \frac{(Fr)(C-Cv^n)}{i}$$

This looks more complicated than the usual formula, but it actually makes this particular problem easier. First, set up the formula for the price of the bond assuming no calls, with a maturity date of 12/31/09:

$$P' = 10,000v^{20} + \frac{10,000(.08)(10,000 - 10,000v^{20})}{10,000(.10)} \\ = 10,000v^{20} + .8(10,000 - 10,000v^{20}) \\ = 8,000 + 2,000v^{20}$$

The point of using Makeham's is the simple resulting price formula. Now, for each of the 20 different maturity dates, the number of years in the original formula would be reduced by 1 year. Allowing for equal probability of maturity at all 20 dates:

$$P = .05(8,000 + 2,000v^{20}) \\ + .05(8,000 + 2,000v^{19}) \\ + \dots \\ + .05(8,000 + 2,000v^1)$$

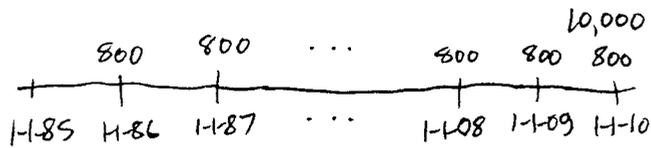
$$P = .05(8,000)20 + .05(2,000)(v^{20} + v^{19} + \dots + v^1) \\ = 8,000 + 100a_{\overline{20}|10\%} \\ = 8,851$$

(B)

The next page shows a solution based on the typical bond price formula.

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(2)



without 5% probabilities, amortized value of bond at 1-1-90 = price at that date to yield 10%

$$= C + C(g-i) a_{\overline{n}|i}$$

$$= 10,000 [1 + (.08-.10) a_{\overline{20}|.10}]$$

with the 5% probabilities, in effect have 20 prices

$$5\% [10,000 (1 + [.08-.10] a_{\overline{17}|.10})$$

$$+ 10,000 (1 + [.08-.10] a_{\overline{21}|.10})$$

$$+ \dots$$

$$+ 10,000 (1 + [.08-.10] a_{\overline{19}|.10})$$

$$+ 10,000 (1 + [.08-.10] a_{\overline{20}|.10})]$$

$$= 10,000 + 10 [a_{\overline{17}|} + a_{\overline{21}|} + \dots + a_{\overline{20}|}]$$

Decreasing annuity

$$= 10,000 + 10 [20 a_{\overline{20}|} - \frac{a_{\overline{20}|} - 20v^{20}}{i}]$$

$$= 10,000 + 10 [170.27 - 55.41]$$

$$= ~~11,119~~ \text{ way outside range!}$$

$$8,851$$

(B)

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$$3. \quad q_{30}^{(2)} = \frac{824}{1,000,000} = .000824 \qquad q_{30}^{(1)} = \frac{2680}{1,000,000} = .00268$$

$$q_{31}^{(2)} = 1.04 q_{30}^{(2)} = .000857$$

$$q_{31}^{(1)} = 1.02 q_{30}^{(1)} = .00273$$

$$q_{32}^{(2)} = 1.04 q_{31}^{(2)} = .000891$$

$$q_{32}^{(1)} = 1.20 q_{31}^{(1)} = .00279$$

$$q_{33}^{(2)} = 1.04 q_{32}^{(2)} = .000927$$

$$q_{33}^{(1)} = 1.02 q_{32}^{(1)} = .00284$$

$${}_3q_{31}^{(2)} = q_{31}^{(2)} + p_{31}^{(T)} q_{32}^{(2)} + p_{31}^{(T)} p_{32}^{(T)} q_{33}^{(2)}$$

= probability that life age 31 succumbs to decrement #2 within 3 years

$$q_{30}^{(3)} = \frac{100,000}{1,000,000} = .10 = q_{31}^{(3)} = q_{32}^{(3)} = q_{33}^{(3)}$$

$$p_{31}^{(T)} = 1 - q_{31}^{(1)} - q_{31}^{(2)} - q_{31}^{(3)} = 1 - .00273 - .000857 - .1 = .89641$$

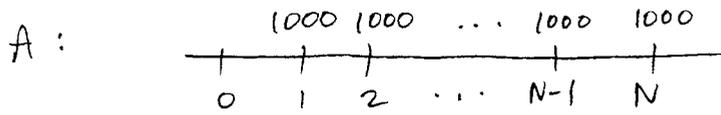
$$p_{32}^{(T)} = 1 - q_{32}^{(1)} - q_{32}^{(2)} - q_{32}^{(3)} = 1 - .00279 - .000891 - .1 = .89632$$

$$\begin{aligned} {}_3q_{31}^{(2)} &= .000857 + .89641(.000891) + .89641(.89632)(.000927) \\ &= .000857 + .000799 + .0007447 \\ &= .00240 \end{aligned}$$

(D)

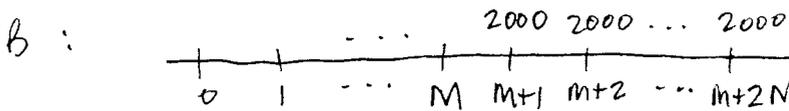
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4.

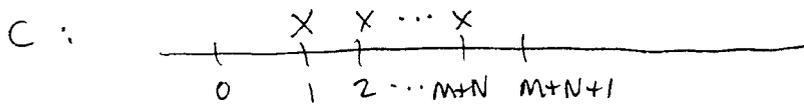


$$a_{\overline{n}|} = 11.101$$

$$A = 1000 a_{\overline{n}|}$$



$$B = 2000 (a_{\overline{M+2N}|} - a_{\overline{M}|})$$



$$C = X (a_{\overline{M+N}|})$$

$$A + B = C$$

$$1000 a_{\overline{n}|} + 2000 (a_{\overline{M+2N}|} - a_{\overline{M}|}) = X a_{\overline{M+N}|}$$

$$X = \frac{1000 a_{\overline{n}|} + 2000 (a_{\overline{M+2N}|} - a_{\overline{M}|})}{a_{\overline{M+N}|}}$$

$$X = \frac{11,101 + 2000 (a_{\overline{M+2N}|} - 8.273)}{12.780}$$

$$v^M a_{\overline{n}|} + a_{\overline{M}|} = a_{\overline{M+N}|} \Rightarrow v^M = \frac{a_{\overline{M+N}|} - a_{\overline{M}|}}{a_{\overline{n}|}} = \frac{12.780 - 8.273}{11.101} = .40600$$

$$v^N a_{\overline{M}|} + a_{\overline{N}|} = a_{\overline{M+N}|} \Rightarrow v^N = \frac{a_{\overline{M+N}|} - a_{\overline{N}|}}{a_{\overline{M}|}} = \frac{12.780 - 11.101}{8.273} = .20295$$

$$a_{\overline{M+N}|} = a_{\overline{M}|} + v^M a_{\overline{n}|} + v^N a_{\overline{N}|}$$

$$= 8.273 + \cancel{.20295} (11.101) (\cancel{1 + .20295}) (4060 + .0824) 11.101$$

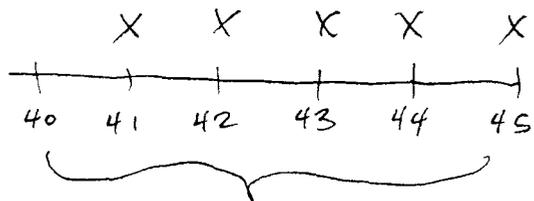
$$= \cancel{10.983} 13.695$$

$$\therefore X = \frac{11,101 + 2000 (\cancel{10.983} 13.695 - 8.273)}{12.780} = \cancel{1295} 1717$$

(C)

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5. 1-1-90 Age 40 Participation begins at 1-1-90  
 since plan is set up 1-1-90



First five years of participation - death benefit is  $1000 \ddot{a}_{57.07} \rightarrow$  be careful that death benefit is paid at the end of the year of death, so annuity timing is  $\ddot{a}$ , not  $a$ !

$$PV = \left( \frac{M_{40} - M_{45}}{D_{40}} \right) 1000 \ddot{a}_{57.07}$$

$$M_x = D_x - d N_x$$

$$M_{40} = 69,444 - .06542 (928,377) = 8,709$$

$$M_{45} = 48,876 - .06542 (625,035) = 7,986$$

$$PV = \left( \frac{8709 - 7986}{69444} \right) 1000 (4.387) = 45.69$$

(D)

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6. Retirement age = 60 Spouse's age = 65

Since the early retirement benefit must be actuarially equivalent, set the PV of early retirement benefits equal to PV of normal ret benefits

$$X (\ddot{a}_{60}^{(12)} + \ddot{a}_{65}^{(12)} - \ddot{a}_{60:65}^{(12)}) = 300 \frac{D_{65}}{D_{60}} \left[ \ddot{a}_{57}^{(12)} + \frac{N_{70}^{(12)}}{D_{65}} \right]$$

$$X = \frac{(300)(D_{65} \ddot{a}_{57}^{(12)} + N_{70}^{(12)})}{\frac{N_{60}^{(12)}}{D_{60}} + \frac{N_{65}^{(12)}}{D_{65}} - \ddot{a}_{60:65}^{(12)}}$$

$$= 300 (97178 (4.254) + 458709) / 148633$$

$$\frac{1458832}{148633} + \frac{848930}{97178} - 7.434$$

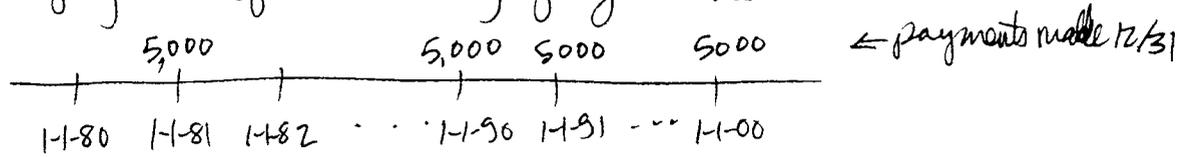
$$= \frac{1760}{9.815 + 8.736 - 7.434}$$

$$= 158.34$$

(D)

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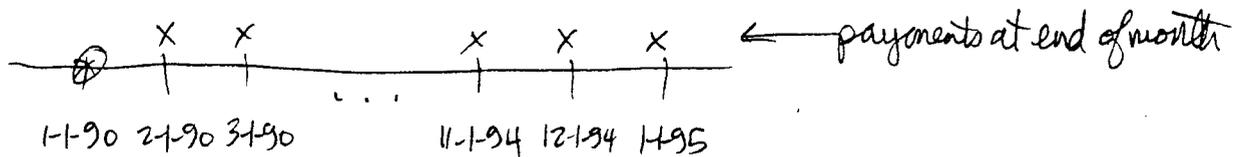
7. At 1-1-90, balance of original loan is simply PV of remaining payments



$$\therefore \text{PV balance} = 5000 a_{\overline{10}|.10} = 30,723$$

For the new loan, calculate monthly payments.

$$7.5\% \text{ per annum} \Rightarrow (1.075)^{\frac{1}{12}} - 1 = .6045\% \text{ per month}$$



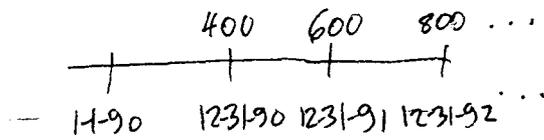
$$30,723 = X a_{\overline{60}|.6045\%}$$

$$X = 612.04$$

(A)

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8. You can directly write down two reversionary annuities for Smith and Brown to provide the desired payments. One trick to this question is that you must be careful how to define the level of the payments:



For Smith, this series of payments is not  $400(a_x - a_{xy}) + 200(Ia_x - Ia_{xy})$  because this expression provides a 600 benefit at 12-31-90!

$$PV = 5000 a_{xy} + 200(a_x - a_{xy}) + 200(Ia_x - Ia_{xy})$$

both alive      Smith only alive = 200 level + 200 tier

$$+ 300(a_y - a_{xy}) + 100(Ia_y - Ia_{xy})$$

Brown only alive = 300 level + 100 tier

$$= 5000(9.4) + 200(13.7 - 9.4) + 200(144.8 - 72.2) + 300(11.694) + 100(107.5 - 72.2)$$

$$= 47,000 + 860 + 14,520 + 660 + 3,530$$

$$= 66,570$$

(A)

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9. Smith age 60, Brown age 55

Probability Smith dies after Brown

$$= q_{55}p_{60} + 1/q_{55} 2p_{60} + 2/q_{55} 3p_{60} + \dots$$

$$= \left( \frac{l_{55} - l_{56}}{l_{55}} \right) \frac{l_{61}}{l_{60}} + \frac{l_{56} - l_{57}}{l_{55}} \left( \frac{l_{62}}{l_{60}} \right) + \frac{l_{57} - l_{58}}{l_{55}} \left( \frac{l_{63}}{l_{60}} \right) + \dots$$

$$= \frac{l_{55}}{l_{55}} \left( \frac{l_{61}}{l_{60}} \right) + \frac{l_{56}}{l_{55}} \left( \frac{l_{62}}{l_{60}} \right) + \frac{l_{57}}{l_{55}} \left( \frac{l_{63}}{l_{60}} \right) - \frac{l_{56}}{l_{55}} \left( \frac{l_{61}}{l_{60}} \right) - \frac{l_{57}}{l_{55}} \left( \frac{l_{62}}{l_{60}} \right) - \frac{l_{58}}{l_{55}} \left( \frac{l_{63}}{l_{60}} \right) \dots$$

$$= \frac{l_{54}}{l_{55}} \left[ \frac{l_{55}}{l_{54}} \left( \frac{l_{61}}{l_{60}} \right) + \frac{l_{56}}{l_{54}} \left( \frac{l_{62}}{l_{60}} \right) + \frac{l_{57}}{l_{54}} \left( \frac{l_{63}}{l_{60}} \right) + \dots \right] - p_{55:60} - 2p_{55:60} - 3p_{55:60} - \dots$$

$$= \frac{1}{p_{54}} \left[ p_{54:60} + 2p_{54:60} + 3p_{54:60} + \dots \right] - e_{55:60}$$

$$= \frac{e_{54:60}}{p_{54}} - e_{55:60}$$

Since  $v^x \ddot{a}_{x+1} = a_x$ , we can eliminate interest and work with expectations:  $p_x(1+e_{x+1}) = e_x$

$$p_{54}(1+e_{55}) = e_{54}$$

$$p_{54} = \frac{22.786}{22.989} = .9912$$

$$\text{Probability} = \frac{14.580}{.9912} - 14.330 = .3799$$

(D)

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10. Desired annuity for Smith and Brown

$$= 1200 \ddot{a}_{\overline{10}|}^{(12)} + \cancel{1200} \ddot{a}_{x+10:y+10}^{(12)} \frac{D_{x+10:y+10}}{D_{xy}} + 600 \ddot{a}_{y+10}^{(12)} \frac{D_{y+10}}{D_y} + 1200 \ddot{a}_{x+10}^{(12)} \frac{D_{x+10}}{D_x}$$

current annuity for Smith and Brown

$$= 1200 \ddot{a}_x^{(12)} + 600 (\ddot{a}_y^{(12)} - \ddot{a}_{xy}^{(12)})$$

Difference between these two is

$$\begin{aligned} & 1200 \ddot{a}_{\overline{10}|}^{(12)} + 1200 \left( \ddot{a}_{x+10}^{(12)} \frac{D_{x+10}}{D_x} - \ddot{a}_x^{(12)} \right) \\ & \quad - 600 \left( \ddot{a}_{x+10:y+10}^{(12)} \frac{D_{x+10:y+10}}{D_{xy}} - \ddot{a}_{xy}^{(12)} \right) \\ & \quad + 600 \left( \ddot{a}_{y+10}^{(12)} \frac{D_{y+10}}{D_y} - \ddot{a}_y^{(12)} \right) \\ & = 1200 \ddot{a}_{\overline{10}|}^{(12)} + 1200 \ddot{a}_{x:\overline{10}|}^{(12)} + 600 \ddot{a}_{xy:\overline{10}|}^{(12)} - 600 \ddot{a}_{y:\overline{10}|}^{(12)} \\ & = 1200 (7.287 - 6.044) + 600 (4.701 - 5.497) \\ & = 1014 \end{aligned}$$

(E)

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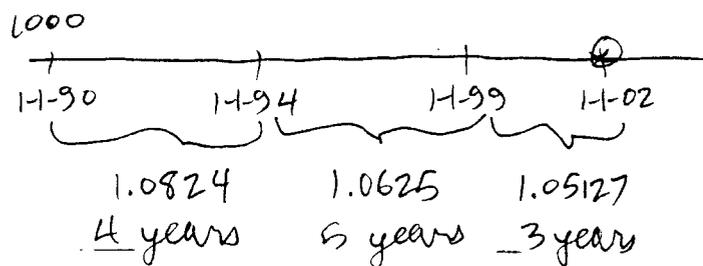
11.

$$\begin{aligned}
 1+i &= \left(1 + \frac{i^{(m)}}{m}\right)^m \\
 &= \left(1 - \frac{d^{(m)}}{m}\right)^{-m} \\
 &= e^{\delta}
 \end{aligned}$$

$$i^{(4)} = .08 \Rightarrow 1+i = (1.02)^4 = 1.0824$$

$$d^{(3)} = .06 \Rightarrow 1+i = (1.02)^3 = 1.0625$$

$$\delta = .05 \Rightarrow 1+i = 1.05127$$



$$\begin{aligned}
 \text{Accumulated value} &= 1000 (1.0824)^4 (1.0625)^5 (1.05127)^3 \\
 &= 2159.51
 \end{aligned}$$

Ⓟ

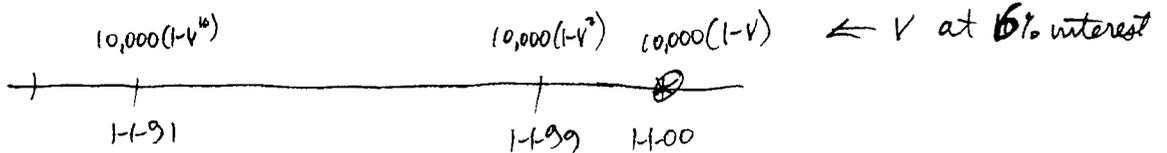
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12.

⊗  $\begin{matrix} 10,000 & 10,000 & \dots & 10,000 \\ | & | & & | \\ 1-90 & 1-91 & 1-92 & 1-00 \end{matrix}$

Original loan amount =  $10,000 a_{\overline{10}|0.06}$

Duration	Payment	Interest	Principal	o/s Loan
0				$10,000 a_{\overline{10} 0.06}$
1	10,000	$10,000(1-v^{10})$	$10,000v^{10}$	$10,000 a_{\overline{9} 0.06}$
2	10,000	$10,000(1-v^9)$	$10,000v^9$	$10,000 a_{\overline{8} 0.06}$
				↓
9	10,000	$10,000(1-v^2)$	$10,000v^2$	$10,000 a_{\overline{1} 0.06}$
10	10,000	$10,000(1-v)$	$10,000v$	$10,000 a_{\overline{0} 0.06}$



Accumulated value at 1-1-2000

$$= 10,000 s_{\overline{10}|0.10} - \frac{10,000}{1.10} \left[ \left( \frac{1.10}{1.06} \right)^{10} + \left( \frac{1.10}{1.06} \right)^9 + \dots + \left( \frac{1.10}{1.06} \right)^2 + \left( \frac{1.10}{1.06} \right)^1 \right]$$

$$= 10,000 s_{\overline{10}|0.10} - \frac{10,000}{1.10} \ddot{s}_{\overline{10}|3.77\%}$$

$$= 10,000 (15.937 - \frac{12.329}{1.10})$$

$$= 47,291$$

ⓑ

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13. 1-1-90 Age 45 → Age 65 at 1-1-2010

12-31-90 contribution =  $.25(50,000) = 12,500$

Max contribution of 30,000 when annual pay is 120,000. Need to know which year this occurs:

$$120,000 = 50,000 (1.05)^n$$

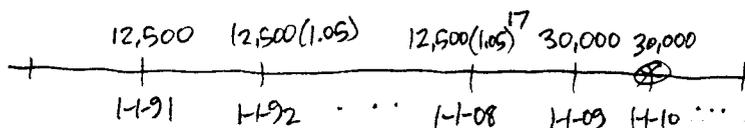
$$2.4 = (1.05)^n$$

$$\log 2.4 = n \log 1.05$$

$$n = \frac{\log 2.4}{\log 1.05} = 17.9$$

1990 compensation = 50,000

2008 compensation =  $50,000 (1.05)^8 = 120,331$



Accumulated value at 1-1-2010

$$= 30,000 + 30,000(1.07) + 12,500(1.05)^{17}(1.07)^2 + \dots + 12,500(1.05)(1.07)^{18} + 12,500(1.07)^1$$

$$= 30,000(2.07) + \frac{12,500}{(1.07)^{19}} \left[ \frac{(1.05)^{17}}{(1.07)^{17}} + \dots + \frac{1.05}{1.07} + 1 \right]$$

$$= 62,100 + 45,207 \ddot{a}_{\overline{18}|1.90\%}$$

$$= 758,566$$

(B)

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$$\begin{aligned} 14. \quad \text{Smith} &= 4000 \ddot{a}_{\overline{50}|0.10} = 4000 (N_{50} - N_{60}) / D_{50} \\ \text{Brown} &= 4000 a_{\overline{50}|0.10} = 4000 (N_{51} - N_{61}) / D_{50} \\ \text{Green} &= 5000 \ddot{a}_{\overline{50}|0.10} \frac{D_{60}}{D_{50}} = 5000 \frac{N_{60}}{D_{50}} \end{aligned}$$

To value Green's annuity, need value of  $D_{50}$

$$\text{Brown} = \text{Smith} - 1975$$

$$4000 \left( \frac{N_{51} - N_{61}}{D_{50}} \right) = 4000 \left( \frac{N_{50} - N_{60}}{D_{50}} \right) - 1975$$

$$4000 (N_{50} - D_{50} - N_{61}) = 4000 (N_{50} - N_{60}) - 1975 D_{50}$$

$$4000 N_{50} - 4000 D_{50} - 4000 N_{61} = 4000 N_{50} - 4000 N_{60} - 1975 D_{50}$$

$$4000 (N_{60} - N_{61}) = 2025 D_{50}$$

$$\begin{aligned} D_{50} &= 4000 \left( \frac{70493 - 63973}{2025} \right) \\ &= 12879 \end{aligned}$$

$$\begin{aligned} \therefore \text{Green's annuity} &= 5000 \left( \frac{70493}{12879} \right) \\ &= 27,367 \end{aligned}$$

(D)

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$$15. \quad A_{60:\overline{1}|} = vP_{60} = .9128$$

$$A_{60} = 1 - d \ddot{a}_{60} = .2895$$

$$A_{61} = 1 - d \ddot{a}_{61} = .3028$$

$$vP_{60} \ddot{a}_{61} = a_{60}$$

$$.2895 = 1 - d \ddot{a}_{60} = 1 - d(a_{60} + 1) = 1 - d(1 + vP_{60} \ddot{a}_{61}) = 1 - d(1 + .9128 \ddot{a}_{61})$$

$$1 - .3028 = d \ddot{a}_{61} \quad \ddot{a}_{61} = \frac{1 - .3028}{d}$$

$$\begin{aligned} &\rightarrow \downarrow \\ .2895 &= 1 - d \left( 1 + \frac{.9128 (.6972)}{d} \right) \\ &= 1 - d - .6364 \end{aligned}$$

$$\begin{aligned} d &= 1 - .2895 - .6364 \\ 1 - v &= .0741 \\ v &= .9259 \end{aligned}$$

$$\begin{aligned} \therefore q_{60} &= 1 - P_{60} \\ &= 1 - \frac{.9128}{v} \\ &= 1 - \frac{.9128}{.9259} \\ &= .01415 \end{aligned}$$

(B)

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16. At 1-1-90, partner age 45  
benefic age 40

$$\begin{aligned}
 PV \text{ net ben} &= v^{20} ({}_{20}p_{45}) ({}_{20}p_{40}) 2400 [ \ddot{a}_{65}^{(12)} + \ddot{a}_{60}^{(12)} - \ddot{a}_{65:60}^{(12)} ] \\
 &+ v^{20} ({}_{20}p_{45}) (1 - {}_{20}p_{40}) 2400 \ddot{a}_{65}^{(12)} \\
 &+ \emptyset \\
 &+ \emptyset \\
 &= (1.05)^{-20} (.83)(.89) 2400 (10 + 11.5 - 6) \\
 &+ (1.05)^{-20} (.83)(.11) 2400 (10) \\
 &= (1.05)^{-20} (.83)(2400) [ .89(15.5) + .11(10) ] \\
 &= (.3769)(.83)(2400)(14.895) \\
 &= 11,183
 \end{aligned}$$

both alive  
benefic dead  
partner dead  
both dead

(3)



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18. On average, death will occur mid-way through the year.

At 1-1-90, participant is age 62

$$\begin{aligned} \text{PV of death benefit} &= v^{\frac{1}{2}} q_{62} [1500(1.09)^{\frac{1}{2}}] \\ &+ {}_1p_{62} v^{\frac{3}{2}} q_{63} [1500(1.09)^{\frac{3}{2}}] \\ &+ {}_2p_{62} v^{\frac{5}{2}} q_{64} [1500(1.09)^{\frac{5}{2}}] \end{aligned}$$

$$v^{\frac{1}{2}} q_{62} = \frac{v^{\frac{62\frac{1}{2}}}}{v^{62} l_{62}} = \frac{\bar{C}_{62}}{D_{62}}$$

$${}_2p_{62} v^{\frac{5}{2}} q_{64} = \frac{l_{64}}{l_{62}} \frac{v^{\frac{64\frac{1}{2}}}}{v^{62}} \frac{d_{64}}{l_{64}} = \frac{\bar{C}_{64}}{D_{62}}$$

$$\therefore \text{PV of death benefit} = 1500 \left[ (1.09)^{\frac{1}{2}} \frac{\bar{C}_{62}}{D_{62}} + (1.09)^{\frac{3}{2}} \frac{\bar{C}_{63}}{D_{62}} + (1.09)^{\frac{5}{2}} \frac{\bar{C}_{64}}{D_{62}} \right]$$

$$= \frac{1500}{13445} [1.044(220.7) + 1.138(223.3) + 1.240(224.4)]$$

(E)

= 85.1 within implied range

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19. 1-1-90 Partic age 65  
Spouse age 63

$$PVB = 12000 \left( \ddot{a}_{65}^{(12)} + .5 \left( \ddot{a}_{63}^{(12)} - \ddot{a}_{65:63}^{(12)} \right) \right) \quad \text{partic for life, spouse after partic dies}$$

$$= 12000 \left[ \frac{N_{65} - \frac{11}{24}}{D_{65}} + .5 \left[ \frac{N_{63} - \frac{11}{24}}{D_{63}} - \frac{N_{65:63} + \frac{11}{24}}{D_{65:63}} \right] \right]$$

$$= 12000 \left[ \frac{893 - \frac{11}{24}}{97} + .5 \left( \frac{1115}{116} - \frac{713}{88} \right) \right]$$

$$= 114,033$$

(C)

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20.  $P = Fr a_{\overline{n}|i} + Cv^n$

10,000 annual coupon = 833.33 per month

monthly interest rate =  $(1.12)^{1/12} = 1.00949$

PV of coupons for any bond at issue =  $833.33 a_{\overline{360}|1.00949\%}$   
= 84,892

PV of coupons for all 40 bonds:

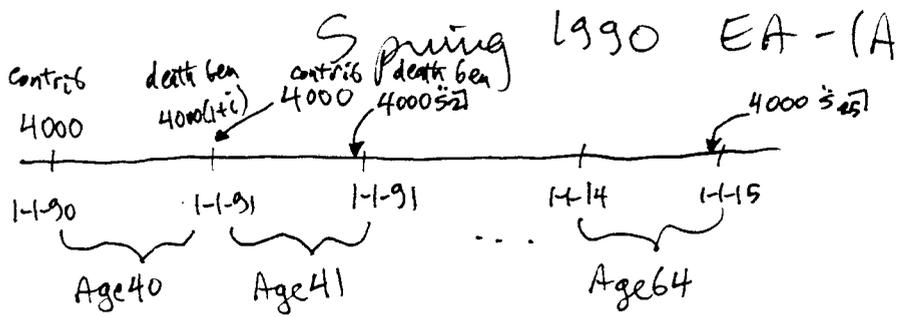
84,892	84,892	...	84,892	84,892
<hr/>				
1-1-90	4-1-90	...	7-1-99	10-1-99

quarterly interest  
=  $(1.12)^{1/4} = 1.0287$

$PV = 84892 \ddot{a}_{\overline{40}|1.0287}$   
= 2,060,484

©

21.



Express this death benefit as a summation:

$$PV = 4000 v^1 q_x \ddot{s}_{\overline{1}|} + 4000 v^2 {}_1|q_x \ddot{s}_{\overline{2}|} + \dots + 4000 v^{25} {}_{24}|q_x \ddot{s}_{\overline{25}|}$$

$$= 4000 (q_{40} \ddot{a}_{\overline{1}|} + {}_1|q_{40} \ddot{a}_{\overline{2}|} + \dots + {}_{24}|q_{40} \ddot{a}_{\overline{25}|})$$

$$= \frac{4000}{l_{40}} (l_{40} - l_{41}) \ddot{a}_{\overline{1}|} + (l_{41} - l_{42}) \ddot{a}_{\overline{2}|} + \dots + (l_{64} - l_{65}) \ddot{a}_{\overline{25}|}$$

$$= \frac{4000}{l_{40}} (l_{40} \ddot{a}_{\overline{1}|} + l_{41} (\ddot{a}_{\overline{2}|} - \ddot{a}_{\overline{1}|}) + l_{42} (\ddot{a}_{\overline{3}|} - \ddot{a}_{\overline{2}|}) + \dots + l_{64} (\ddot{a}_{\overline{25}|} - \ddot{a}_{\overline{24}|}) + l_{65} \ddot{a}_{\overline{25}|})$$

$$= \frac{4000}{l_{40}} (l_{40} + v l_{41} + v^2 l_{42} + \dots + v^{24} l_{64} - l_{65} v^{25} \ddot{s}_{\overline{25}|})$$

$$= \frac{4000}{v^{40} l_{40}} (v^{40} l_{40} + v^{41} l_{41} + v^{42} l_{42} + \dots + v^{64} l_{64} - v^{65} l_{65} \ddot{s}_{\overline{25}|})$$

$$= \frac{4000}{D_{40}} (D_{40} + D_{41} + \dots + D_{64} - D_{65} \ddot{s}_{\overline{25}|})$$

$$= \frac{4000}{D_{40}} (N_{40} - N_{65} - \ddot{s}_{\overline{25}|} D_{65})$$

$$= 4000 \left( \frac{3,524,127 - 422,962 - 58.156(422,962 - 378,659)}{3,524,127 - 3,284,160} \right)$$

$$= 8746$$

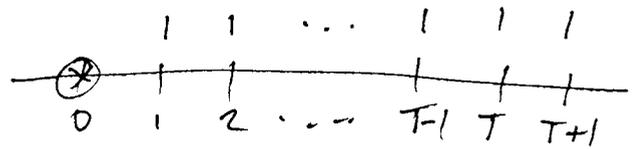
(E)

The key to working this problem is that you can't use  $M_x$  or  $R_x$  functions to value an increasing death benefit of  $4000 \ddot{s}_{\overline{t}|}$  each year. The only hope is to express things in simplest terms, and hope that there is some cancellation.

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22  $\ddot{a}_{\overline{T}|} = 7.452$

$\ddot{a}_{\overline{T+1}|} = 7.950$



$$\begin{aligned} v(\ddot{a}_{\overline{T}|}) + 1 &= \ddot{a}_{\overline{T+1}|} \\ v &= \frac{\ddot{a}_{\overline{T+1}|} - 1}{\ddot{a}_{\overline{T}|}} \\ &= \frac{7.95 - 1}{7.452} \\ &= .932636 \\ (1+i) &= 1.07223 \end{aligned}$$

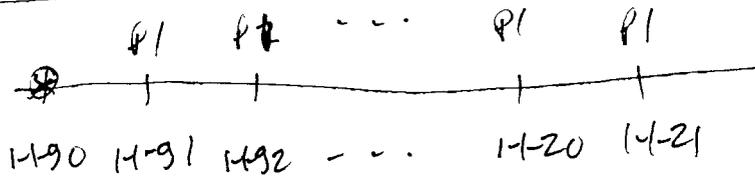
Ⓟ

$\ddot{s}_{\overline{25}|i} = 70.0285$

This is one of the few identity questions on this exam that is relatively easy!

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23. Easiest way to work this for me is to convert nominal rates to annual effective rate - this coincides with the annual payment frequency  
original loan



$$100,000 = P1 a_{\overline{30}|i}$$

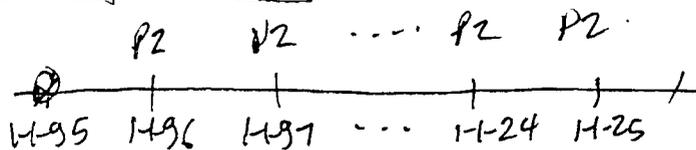
$i$  is annual equivalent to 10% compounded semiannually,  
 $i = (1 + \frac{10}{2})^2 - 1 = 1.1025 - 1 = 0.25\%$

O/S balance at 1-95 without the 5000 additional payment at 1-94 is  
 $O/S = P1 (a_{\overline{25}|0.25\%})$

with the additional payment, we have

$$\begin{aligned} O/S \text{ balance} &= P1 (a_{\overline{25}|0.25\%}) - 1.1025(5000) \\ &= 100,000 \left( \frac{a_{\overline{25}|}}{a_{\overline{30}|}} \right) - 5513 \\ &= 90,930 \end{aligned}$$

Renegotiated loan



$90,930 = P2 a_{\overline{30}|j}$  where  $j$  is annual rate equivalent to 8% compounded semiannually  $\Rightarrow j = (1.04)^2 - 1 = 8.16\%$

$$O/S \text{ loan balance} = P2 (a_{\overline{30}|j}) = 90,930 \left( \frac{a_{\overline{5}|}}{a_{\overline{30}|}} \right) = 32,600$$

(B)

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24.

Probability = Prob (both age 60,65 survive 10 years)

<sup>times</sup>  
[1 - Prob (both age 70,75 survive 5 years)]

$$= {}_{10}P_{60} ({}_{10}P_{65}) [1 - {}_5P_{70} ({}_5P_{75})]$$

$$= .73 (.706) [1 - .74 (.72)]$$

$$= .5154 (.4672)$$

$$= .2408$$

(B)

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25. Net premium reserve = PVFB - PVF premiums

$$\begin{aligned} \text{At age 60, net reserve} &= 1000 A_{60} - 16.61529 \left( \frac{N_{60}}{D_{60}} \right) \\ &= 637.795 - 16.61529 \left( \frac{16,084,767}{1,293,434} \right) \\ &= 431.172 \end{aligned}$$

$$CSV_{60} = {}_tV_{60-t} - (\text{expenses}) \left( \frac{N_{60} - N_{65}}{D_{60}} \right)$$

$$\begin{aligned} \therefore \text{Expenses} &= \frac{{}_tV_{60-t} - CSV_{60}}{\left( \frac{N_{60} - N_{65}}{D_{60}} \right)} \\ &= \frac{431.172 - 423.19}{\left[ \frac{16,084,767 - 10,253,150}{1,293,434} \right]} \\ &= 1.77 \end{aligned}$$

$$CSV_{62} = {}_tV_{62-t} - (\text{expenses}) \left( \frac{N_{62} - N_{65}}{D_{62}} \right)$$

$$\begin{aligned} &= 1000 A_{62} - 16.61529 \left( \frac{N_{62}}{D_{62}} \right) - 1.77 \left( \frac{N_{62} - N_{65}}{D_{62}} \right) \\ &= 661.069 - 16.61529 \left( \frac{13,562,294}{1,165,483} \right) - 1.77 \left( \frac{13,562,294 - 10,253,150}{1,165,483} \right) \\ &= 462.69 \end{aligned}$$

(C)

I'm not 100% sure of the interpretation of the CSV at ages 60 and 62. The expense charge is level over the period from issue age to 65. We don't know the issue age - but for a prospective reserve definition it makes sense to use  $\ddot{a}_{x:\overline{65}|}$  (expenses).